Principles of Communications
ECS 332

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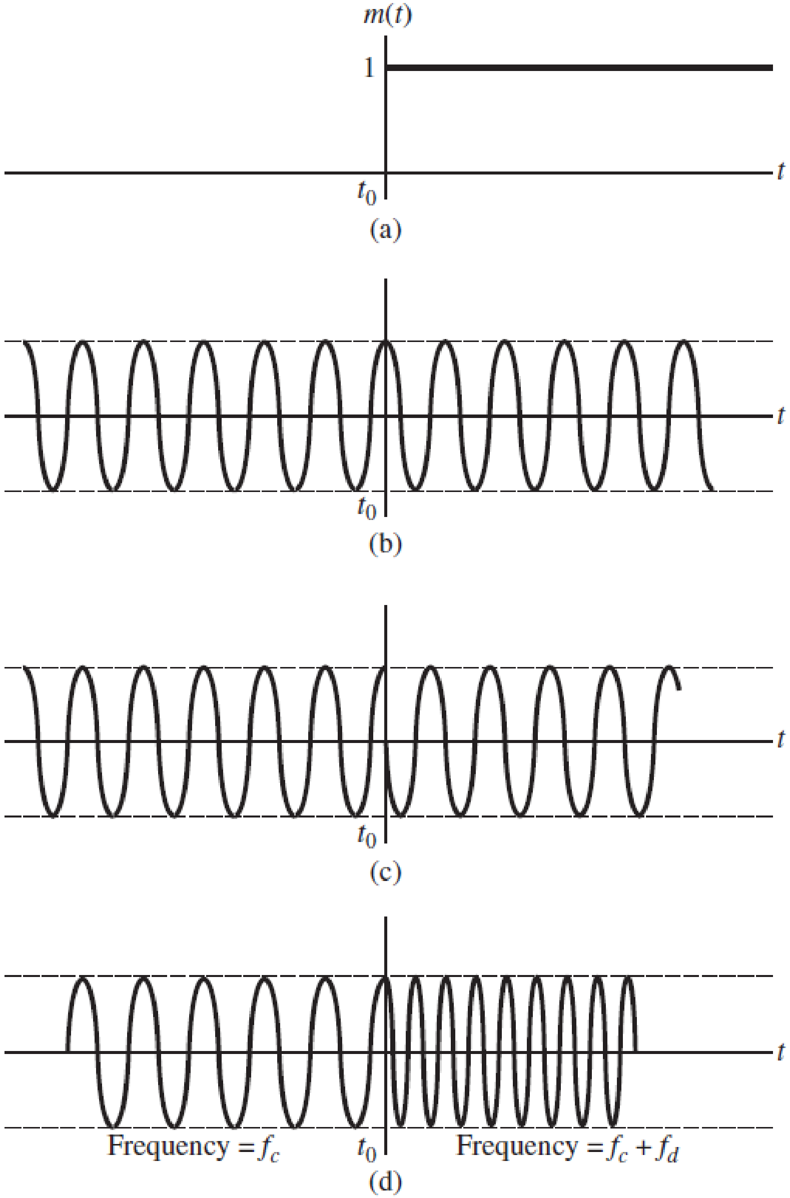
5. Angle Modulation

Office Hours:
BKD, 4th floor of Sirindhralai building

Monday	9:30-10:30
Monday	14:00-16:00
Thursday	16:00-17:00

FM vs. PM

Figure 25



$m(t)$

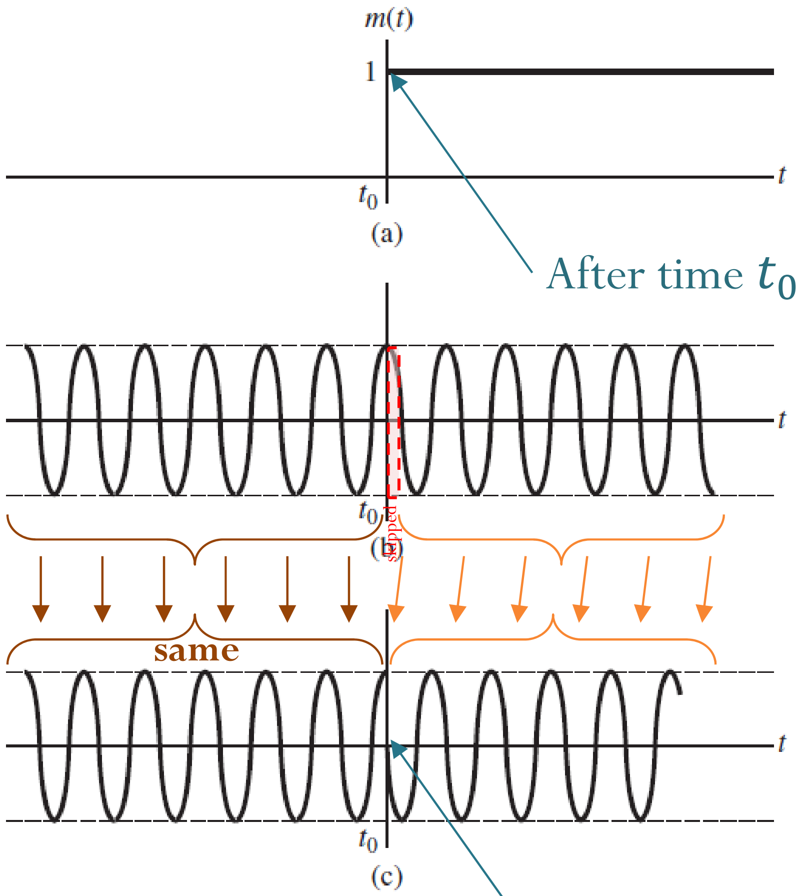
$A\cos(2\pi f_c t + \phi)$

$x_{\mathrm{PM}}(t)$

$x_{\mathrm{FM}}(t)$

Phase Modulation

Figure 25



$$m(t) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases}$$

After time t_0, the message jumps to the value 1.

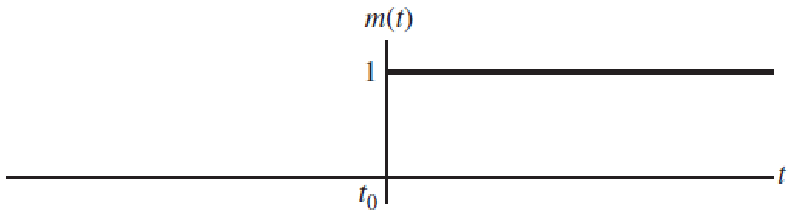
$$A\cos(2\pi f_c t + \phi)$$

$$x_{\mathrm{PM}}(t) = \begin{cases} A\cos(2\pi f_c t + \phi + \mathbf{0^\circ}), & t < t_0 \\ A\cos(2\pi f_c t + \phi + \mathbf{90^\circ}), & t > t_0 \end{cases}$$

After time t_0, the phase is skipped ahead (advanced) by $\mathbf{90^\circ}$.

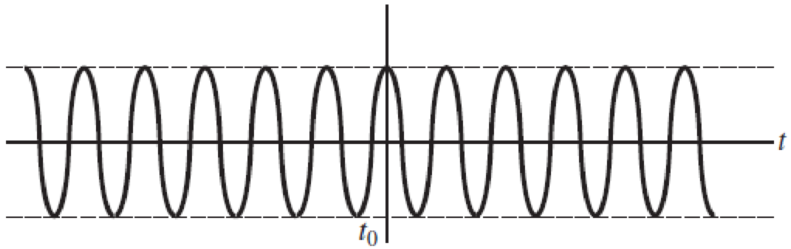
Frequency Modulation

Figure 25



(a)

$m(t)$

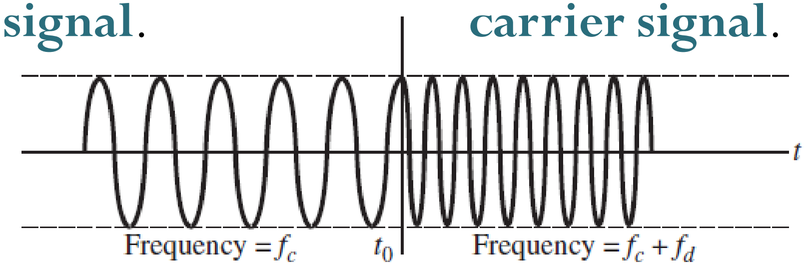


(b)

$A\cos(2\pi f_c t + \phi)$

When the message $\boldsymbol{m(t)}$ is 0, the $\boldsymbol{x_{\mathrm{FM}}(t)}$ has the **same frequency** as **the carrier signal**.

When the message $\boldsymbol{m(t)}$ is 1, the **frequency** of $\boldsymbol{x_{\mathrm{FM}}(t)}$ is higher than the frequency of **the carrier signal**.

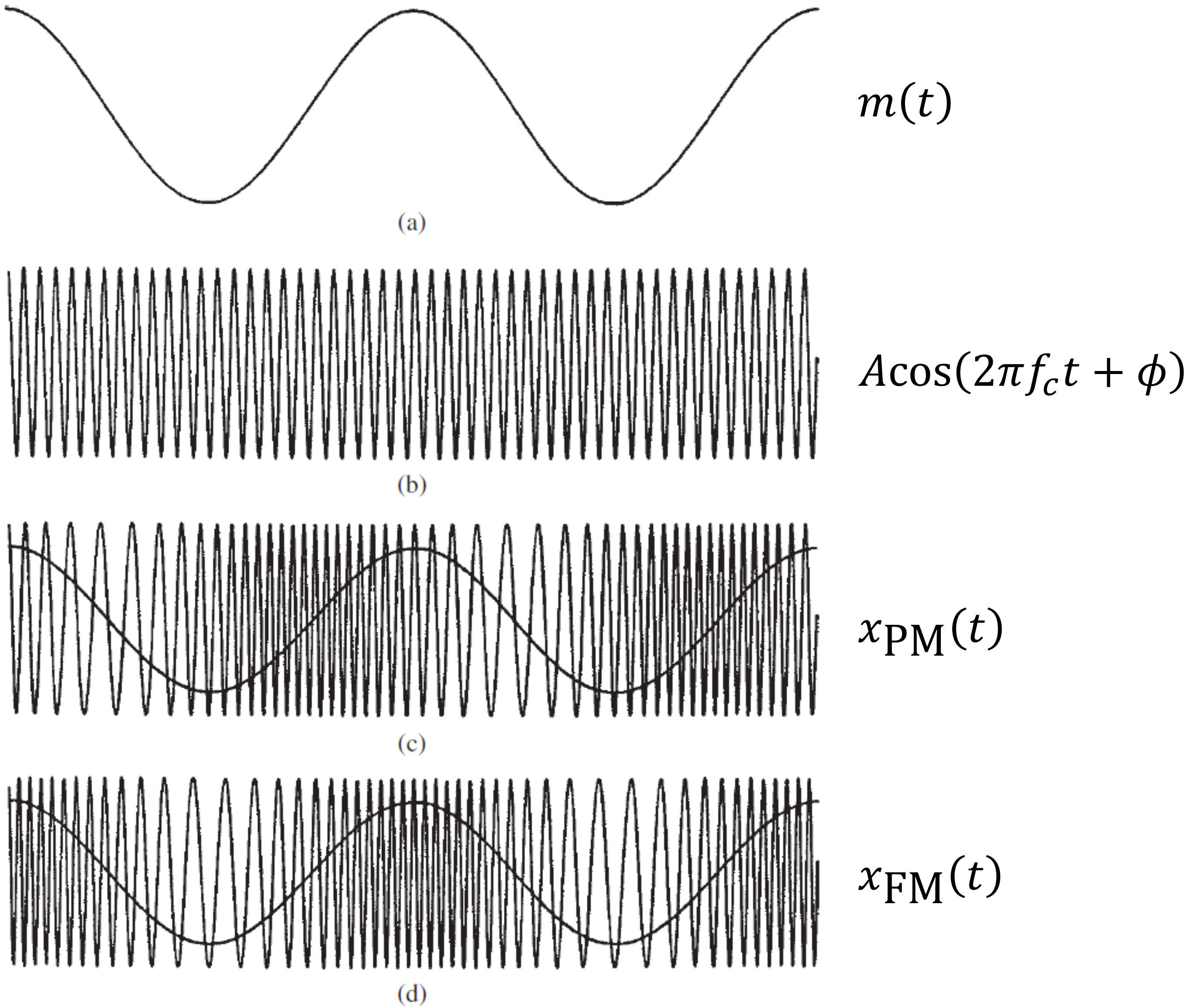


(d)

$x_{\mathrm{FM}}(t)$

FM vs. PM

Figure 24



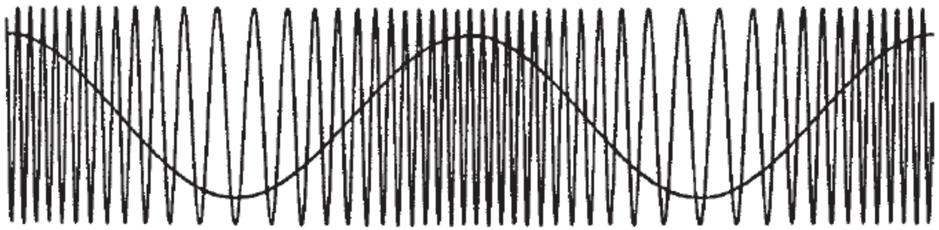
Frequency Modulation

Figure 24

$m(t)$

(a)

It should be evident that the frequency is changing.

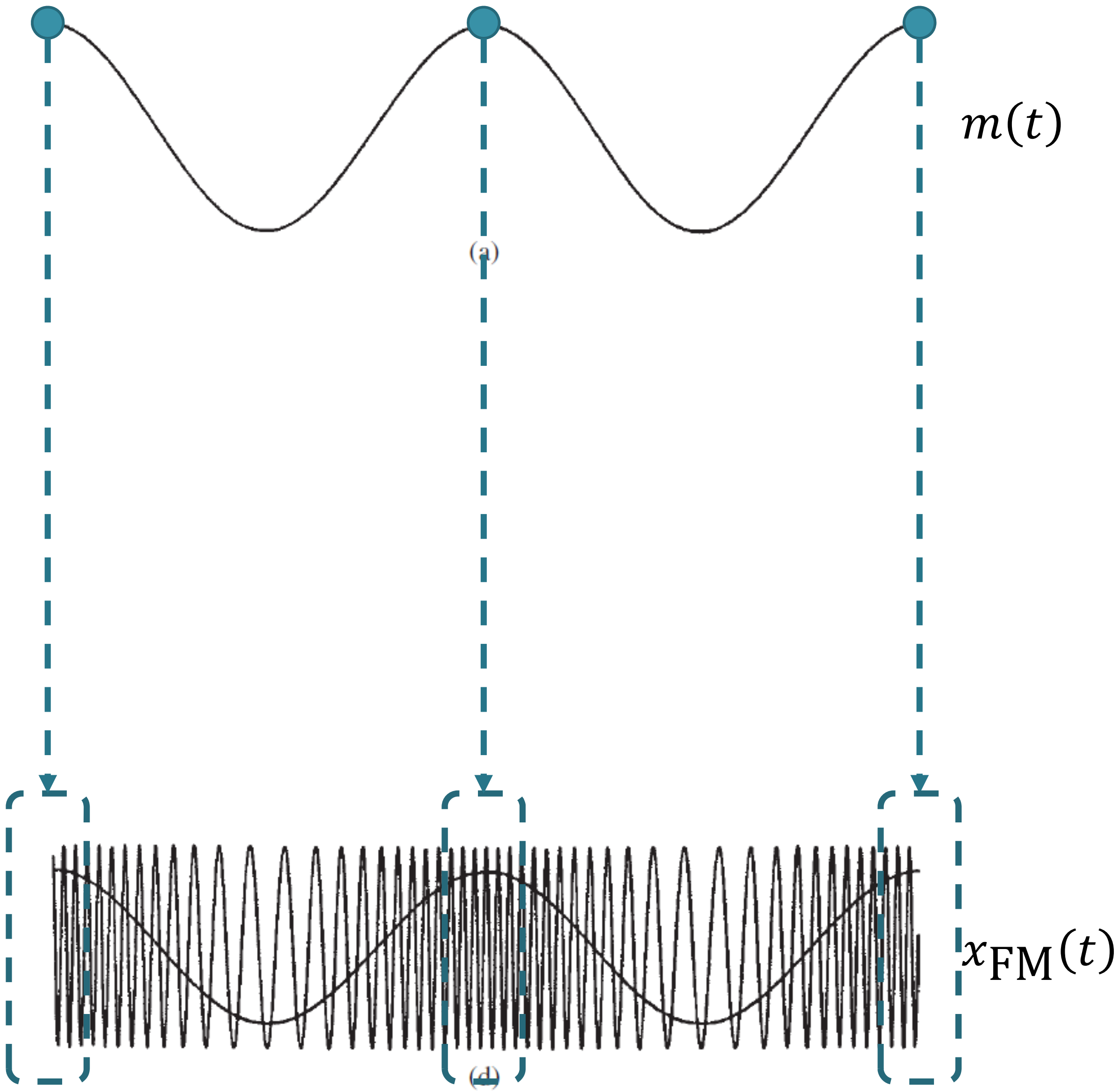


$x_{\mathrm{FM}}(t)$

(d)

Frequency Modulation

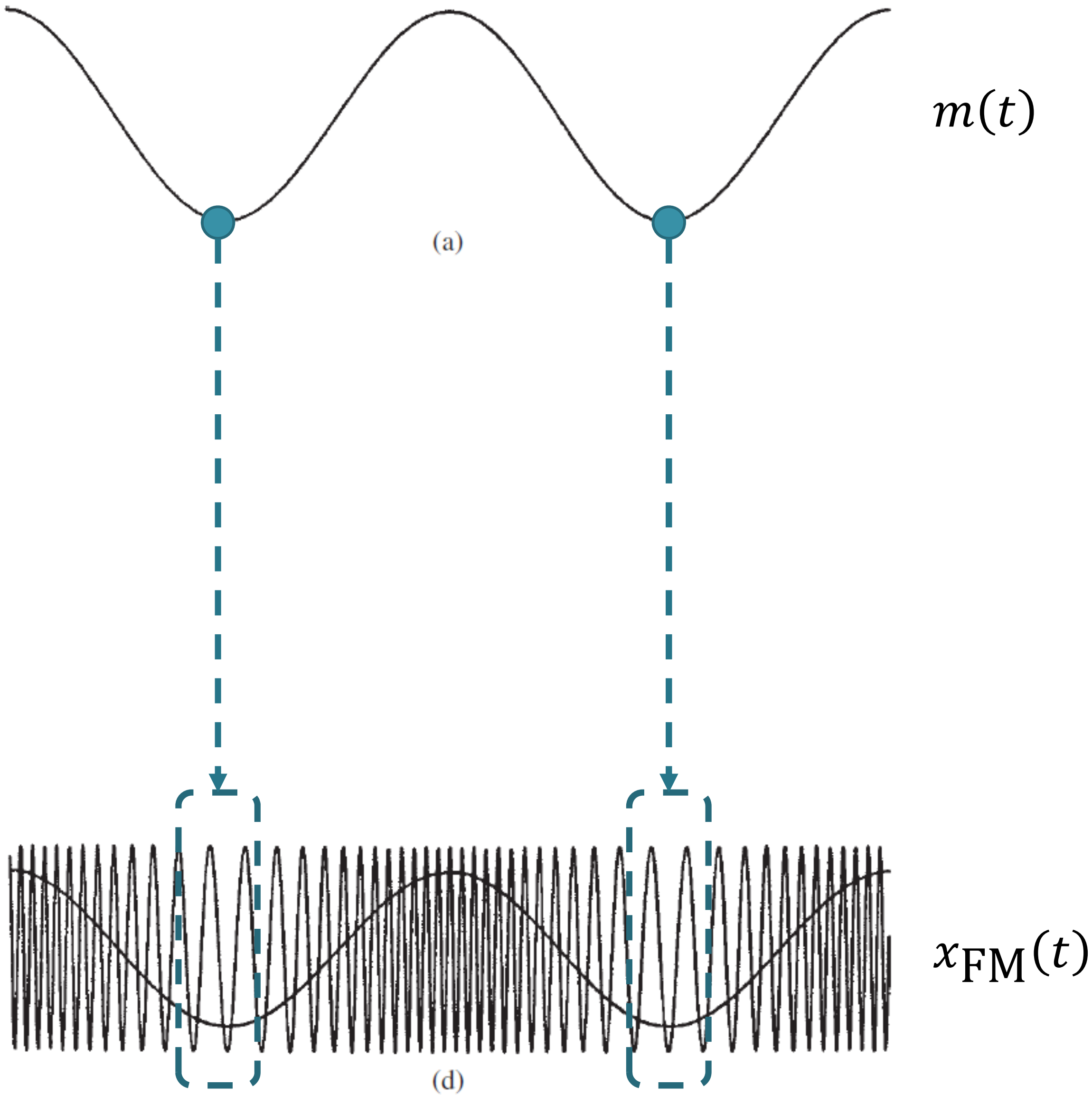
Figure 24



The time at which $\boldsymbol{m(t)}$ is at its **maximum** value corresponds to the time at which $\boldsymbol{x_{FM}(t)}$ has **maximum frequency**.

Frequency Modulation

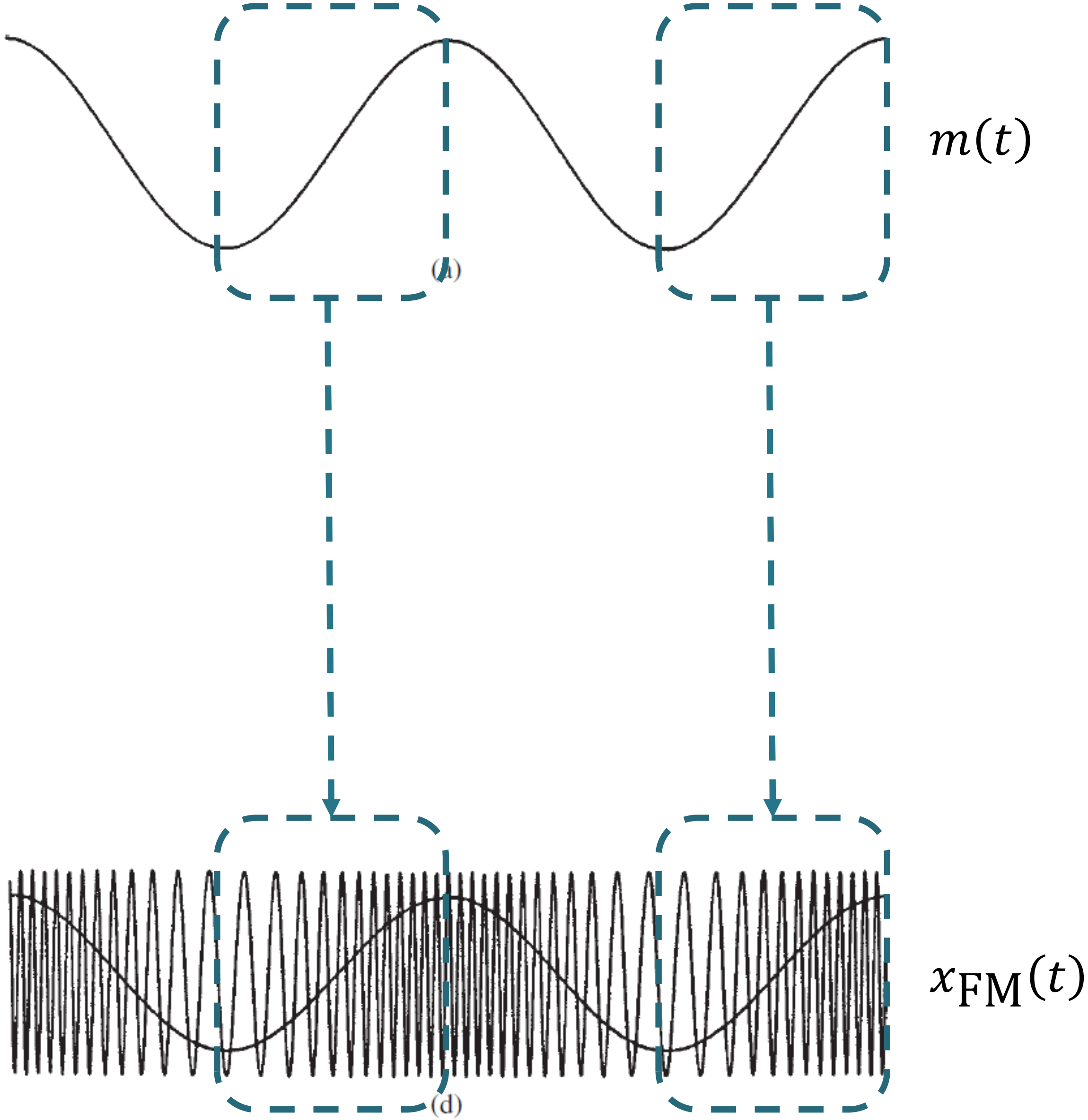
Figure 24



The time at which $\boldsymbol{m(t)}$ is at its **minimum** value corresponds to the time at which $\boldsymbol{x_{FM}(t)}$ has **minimum frequency**.

Frequency Modulation

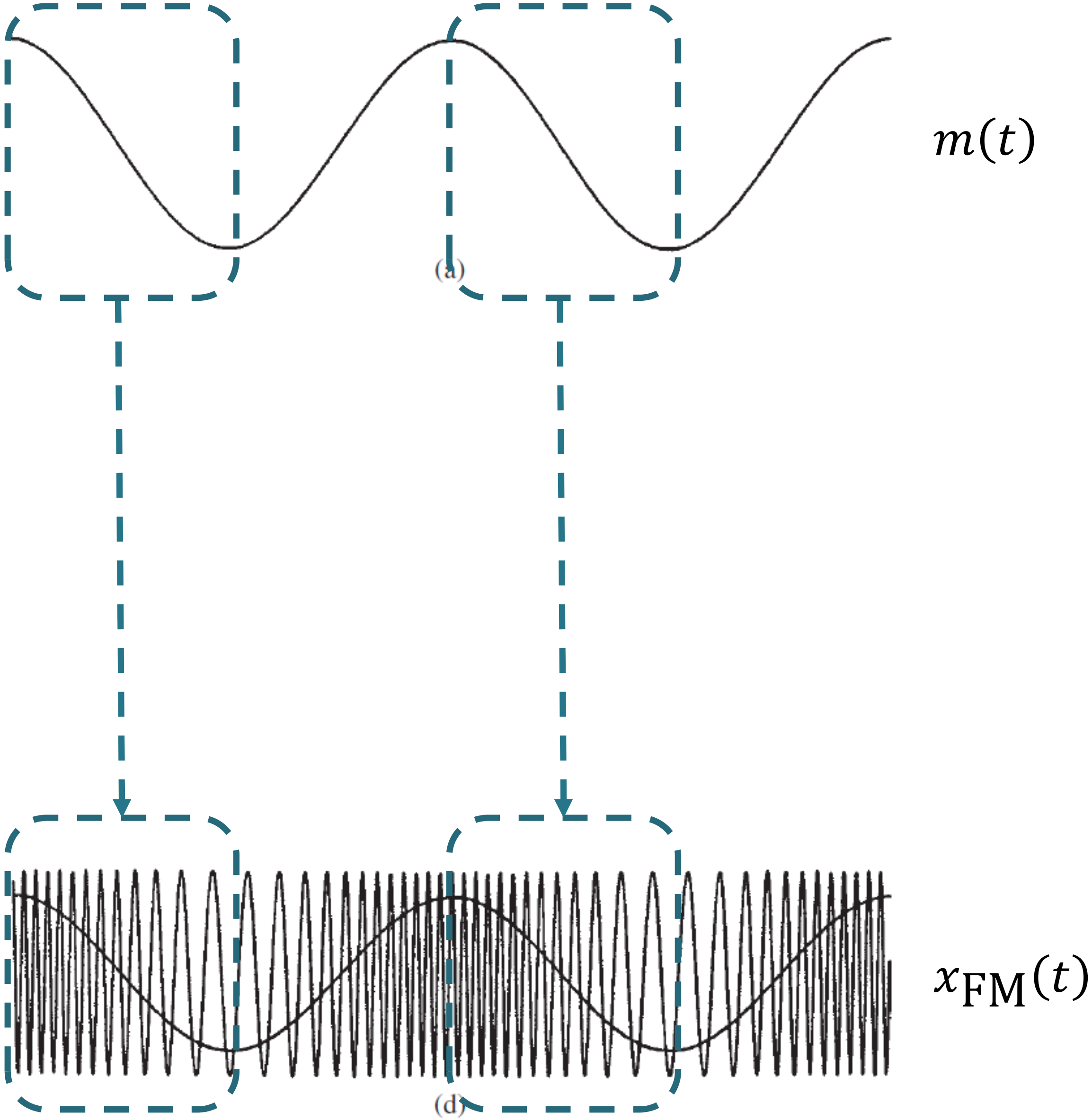
Figure 24



The time interval during which $\boldsymbol{m(t)}$ is **increasing** corresponds to the time interval during which $\boldsymbol{x_{FM}(t)}$ has **increasing frequency**.

Frequency Modulation

Figure 24



The time interval during which $\boldsymbol{m(t)}$ is **decreasing** corresponds to the time interval during which $\boldsymbol{x_{FM}(t)}$ has **decreasing frequency**.

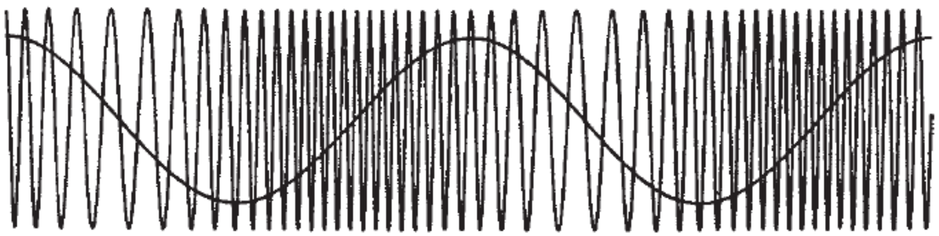
Phase Modulation

Figure 24

$m(t)$

(a)

In $x_{\mathrm{PM}}(t)$, the **phase** varies in proportion with $m(t)$.
When $m(t)$ and hence the phase of $x_{\mathrm{PM}}(t)$ change **continuously**, it is difficult to see the connection with the actual plot of $x_{\mathrm{PM}}(t)$.



$x_{\mathrm{PM}}(t)$

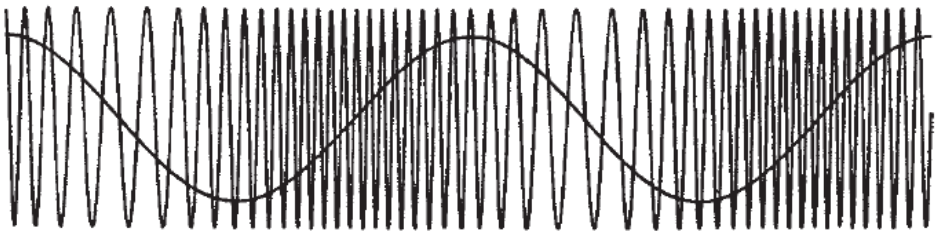
(c)

Phase Modulation

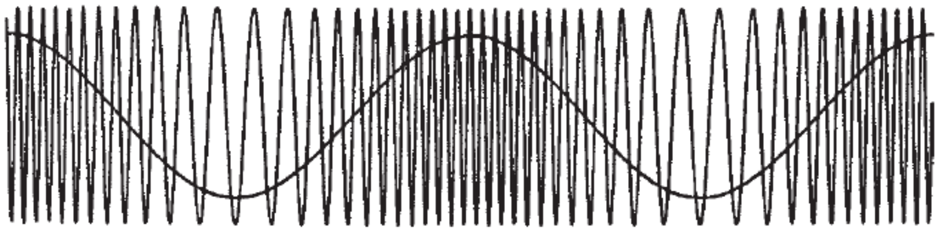
Figure 24

(a)

One may notice here that, in this example, $x_{\mathrm{PM}}(t)$ is similar to $x_{\mathrm{FM}}(t)$ Except that the graph is shifted. However, it is still not clear (visually) how the graph of $x_{\mathrm{PM}}(t)$ is related to $m(t)$.



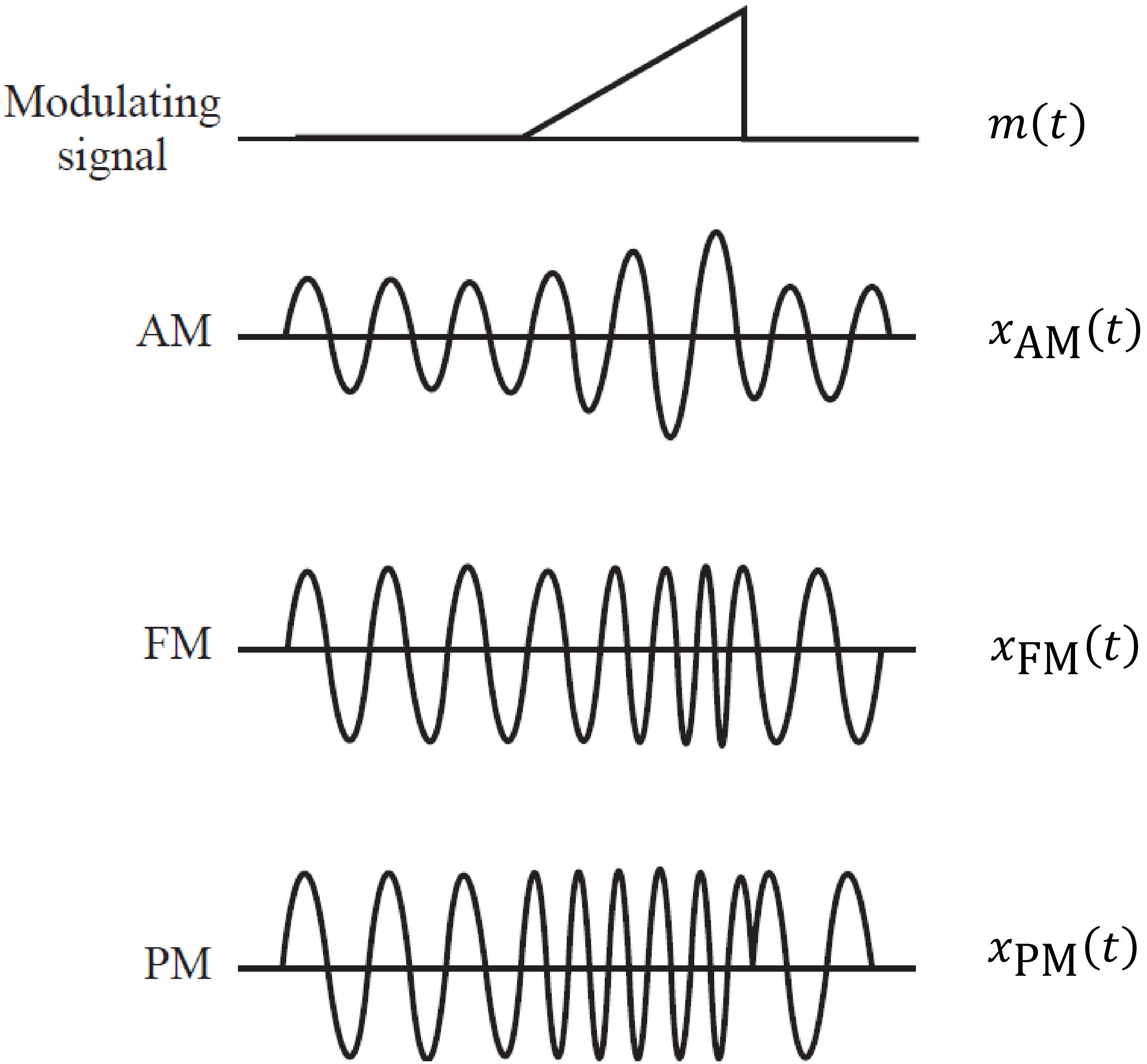
(c)



(d)

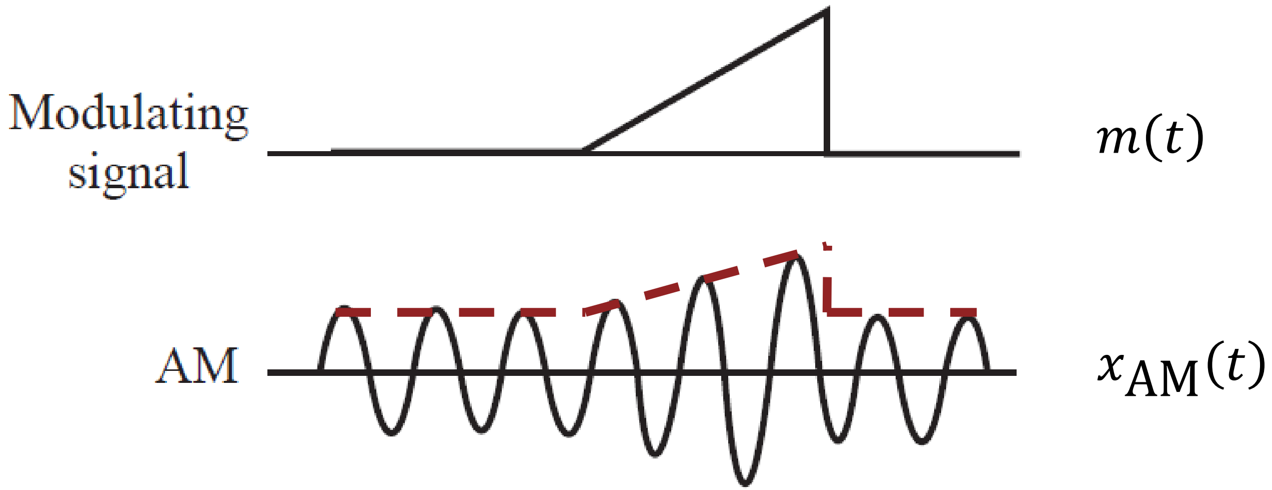
AM, FM, and PM

Figure 26



Amplitude Modulation

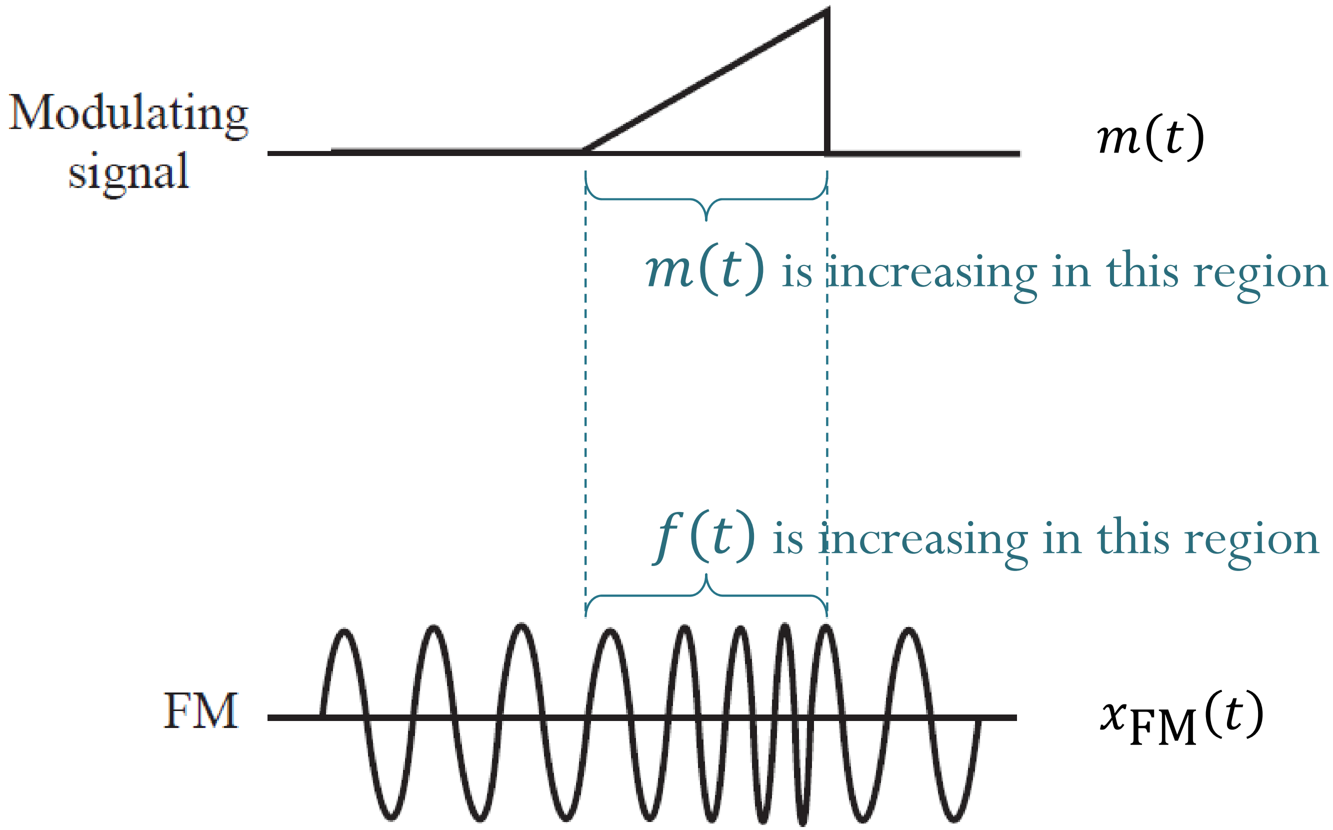
Figure 26



In $x_{AM}(t)$, the **envelope** varies in proportion with $m(t)$.

Frequency Modulation

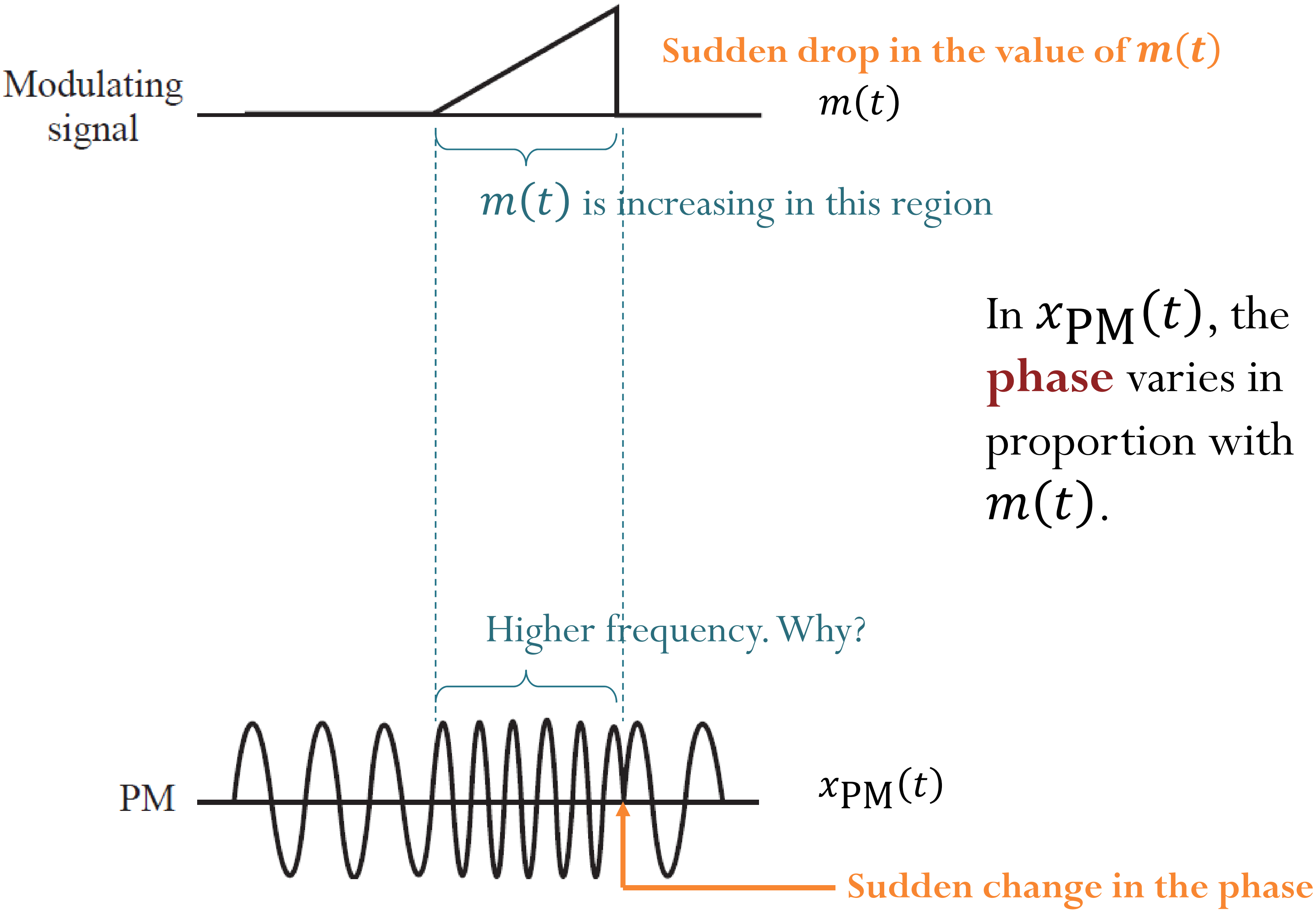
Figure 26



In $x_{FM}(t)$, the **frequency** varies in proportion with $m(t)$.

Phase Modulation

Figure 26



In $x_{\mathrm{PM}}(t)$, the **phase** varies in proportion with $m(t)$.

Instantaneous Frequency

- Sinusoidal signal:

$$g(t) = A\cos(2\pi \boldsymbol{f_0} t + \phi)$$

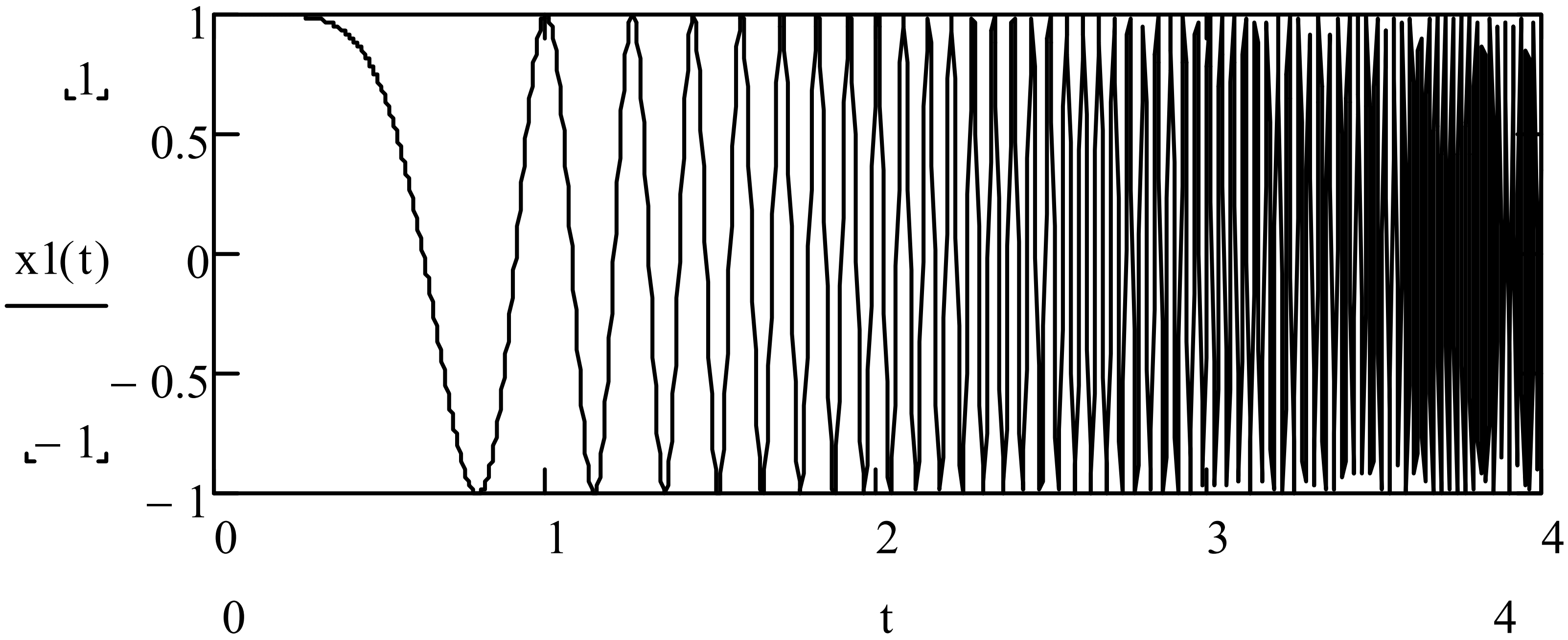
  - Frequency $= \boldsymbol{f_0}$
- Generalized sinusoidal signal:

$$g(t) = A\cos\big(\boldsymbol{\phi(t)}\big)$$

  - Frequency = ?
    - Observation: Frequency value may vary as a function of time.
      - "**instantaneous frequency**"
- Why do we need to find the instantaneous frequency?
  - Analyze Doppler effect (or Doppler shift)
  - Implement **frequency modulation** (**FM**)
    - where the instantaneous frequency will follow the message $m(t)$.

Instantaneous Frequency

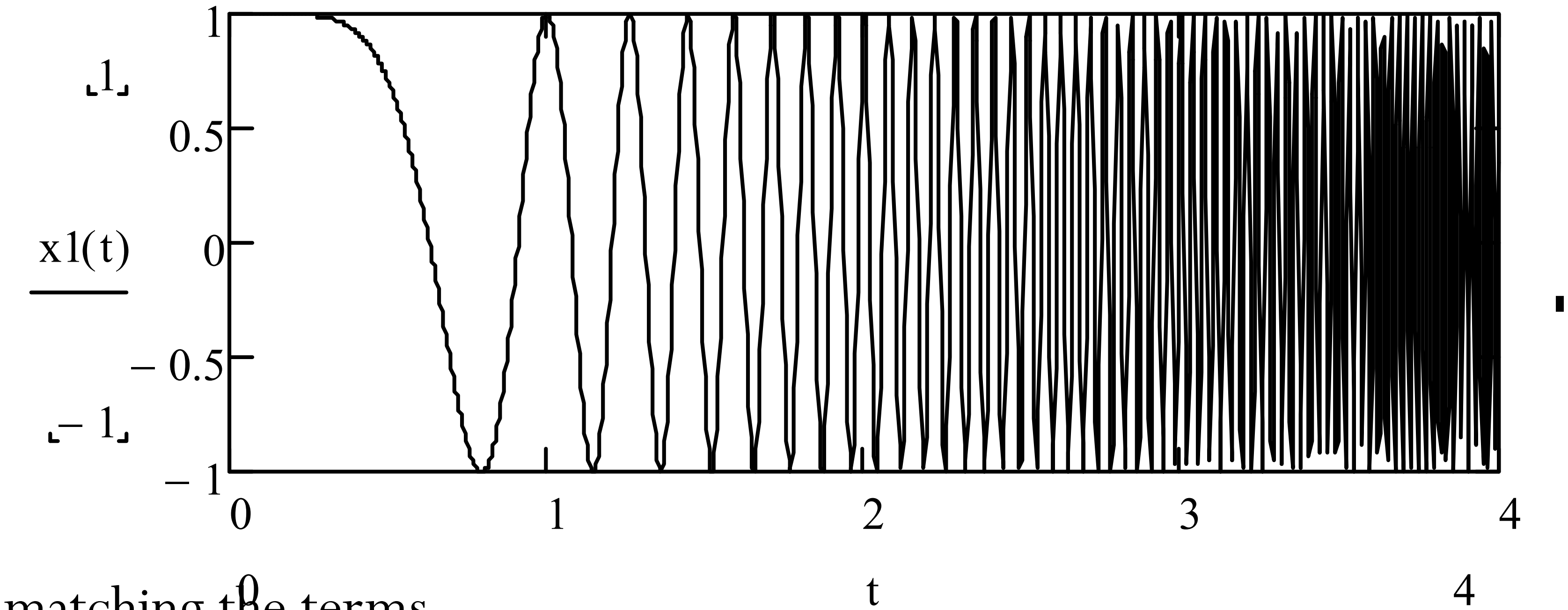
$$x_1(t) = \cos\left(2\pi t^2 t\right)$$



At t = 2, frequency = ?

Instantaneous Frequency

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$



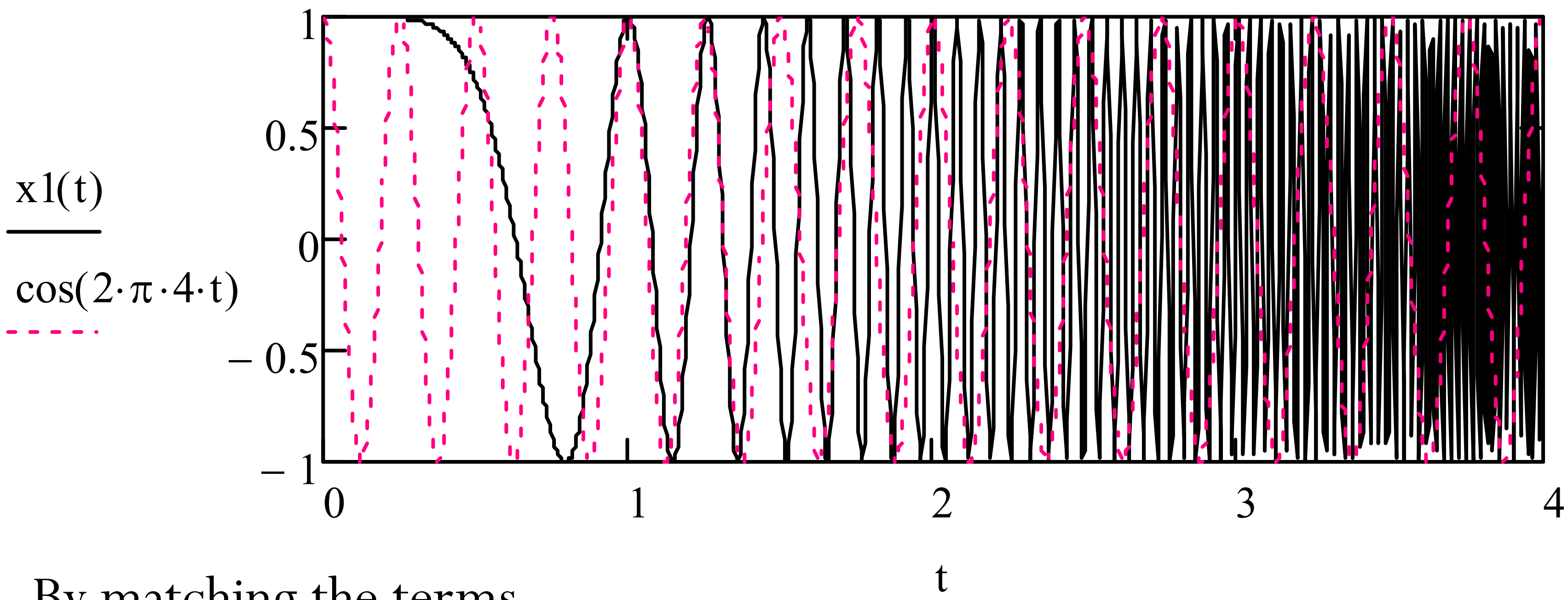
By matching the terms with $\cos(2\pi \boldsymbol{f_0} t)$, you may guess that $\boldsymbol{f(t)} = t^2$.

At t = 2, $f = t^2 = 4$ Hz?

Correct?

Instantaneous Frequency

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$



By matching the terms with $\cos(2\pi \boldsymbol{f_0} t)$, you may guess that $\boldsymbol{f}(\boldsymbol{t}) = t^2$.

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Instantaneous Frequency

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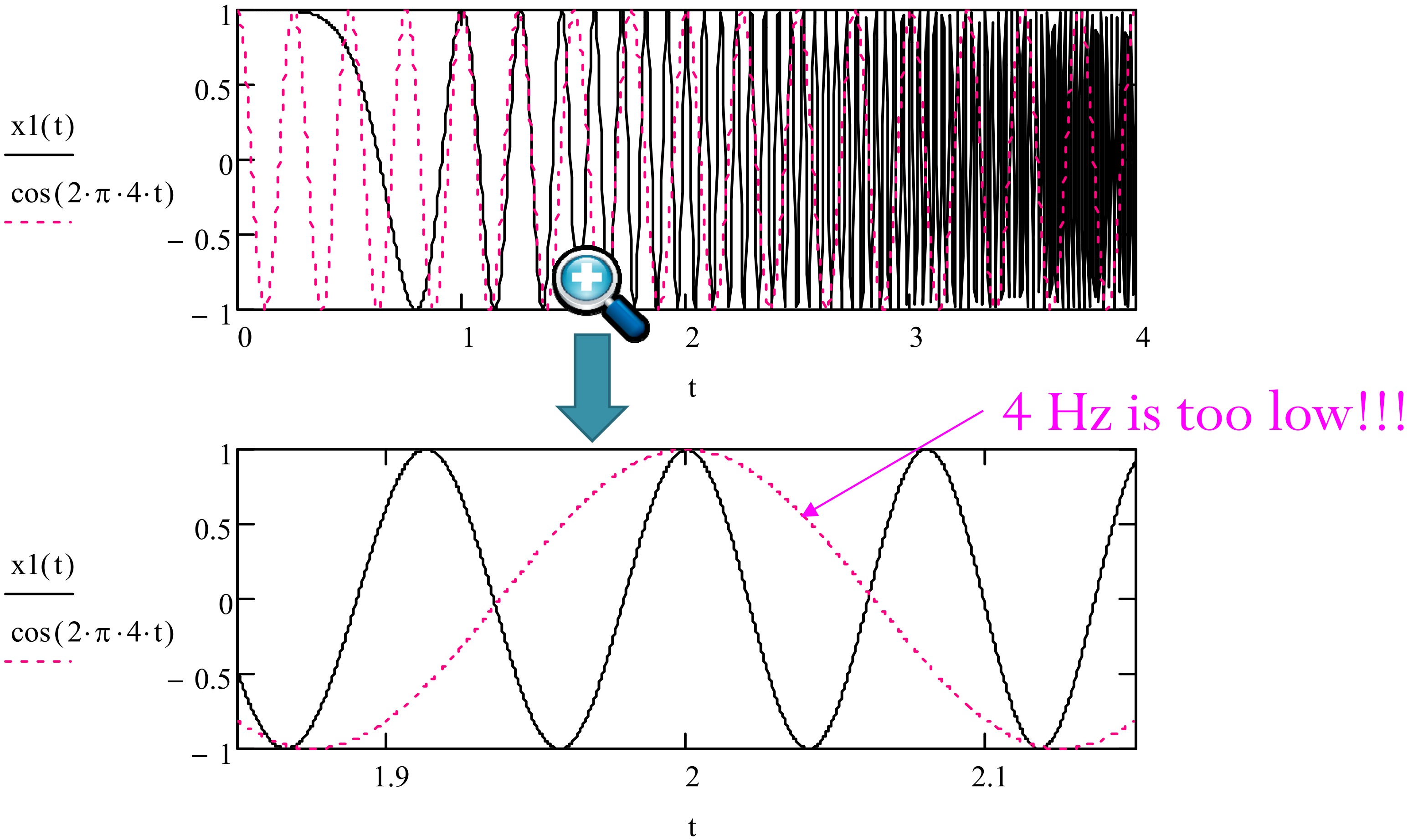
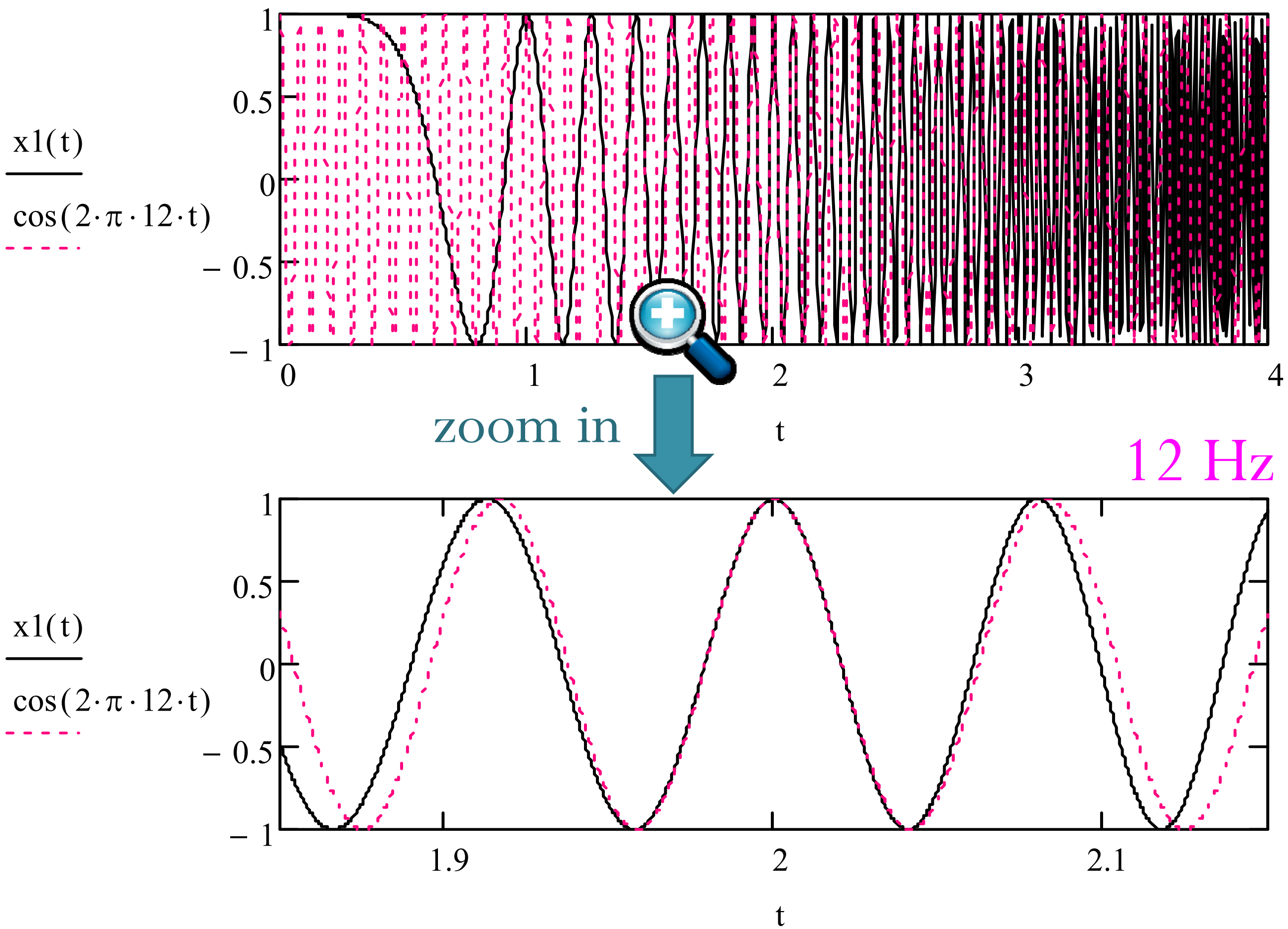


Instantaneous Frequency

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$



12 Hz fits better.

12 Hz?

Instantaneous Frequency

- Sinusoidal signal:

$$g(t) = A\cos(2\pi \boldsymbol{f_0} t + \phi)$$

  - Frequency $= \boldsymbol{f_0}$

- Generalized sinusoidal signal:

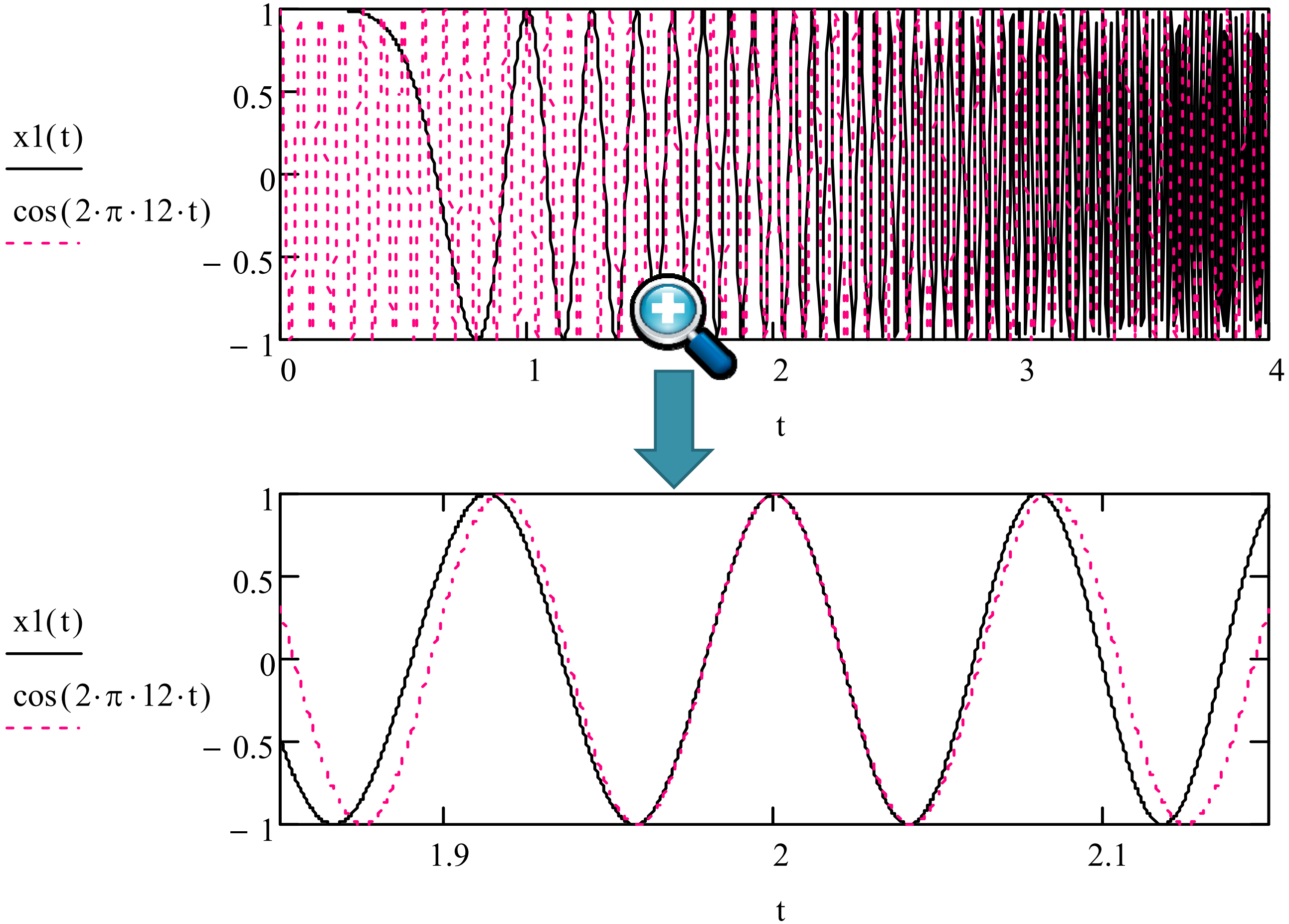
$$g(t) = A\cos\big(\boldsymbol{\phi(t)}\big)$$

  - The **instantaneous frequency** at time *t* is given by

$$\boldsymbol{f(t)} = \frac{1}{2\pi}\frac{d}{dt}\boldsymbol{\phi(t)}$$

Example

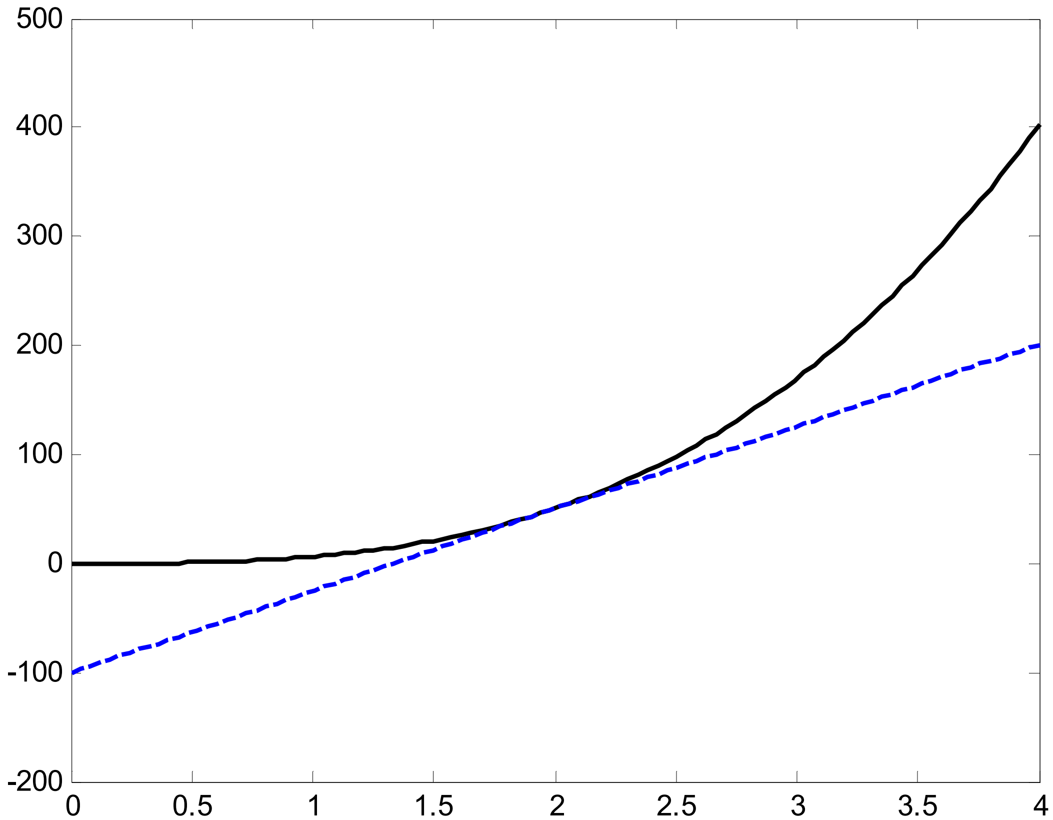
$$x_1(t) = \cos\left(2\pi t^2 t\right)$$



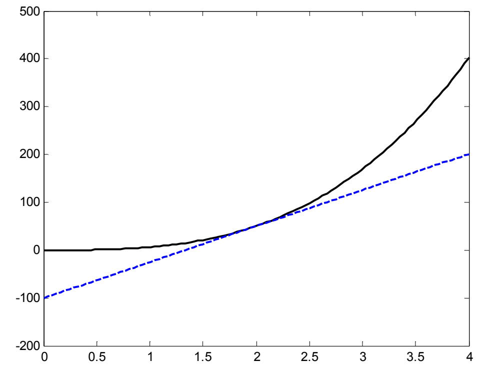
$$f(t) = \frac{1}{2\pi}\frac{d}{dt}\phi(t) = \frac{1}{\cancel{2\pi}}\frac{d}{dt}\left(\cancel{2\pi} t^2 t\right) = 3t^2 \Rightarrow f(2) = 3\times 2^2 = 12$$

First-order (straight-line) approximation/linearization

- How does the formula $\boldsymbol{f(t)} = \frac{1}{2\pi}\frac{d}{dt}\boldsymbol{\phi(t)}$ work?
- Technique from Calculus: first-order (tangent-line) approximation/linearization



First-order (straight-line) approximation/linearization



- How does the formula $\boldsymbol{f(t)} = \frac{1}{2\pi}\frac{d}{dt}\boldsymbol{\phi(t)}$ work?
- Technique from Calculus: first-order (tangent-line) approximation/linearization
  - When we consider a function $\boldsymbol{\phi(t)}$ near a particular time, say, $t = t_0$, the value of the function is approximately

$$\phi(t) \approx \underbrace{\phi'(t_0)}_{\text{slope}}(t - t_0) + \phi(t_0) = \underbrace{\phi'(t_0)}_{\text{slope}} t + \underbrace{\phi(t_0) - t_0\phi'(t_0)}_{\text{constant}}$$

  - Therefore, near $t = t_0$,

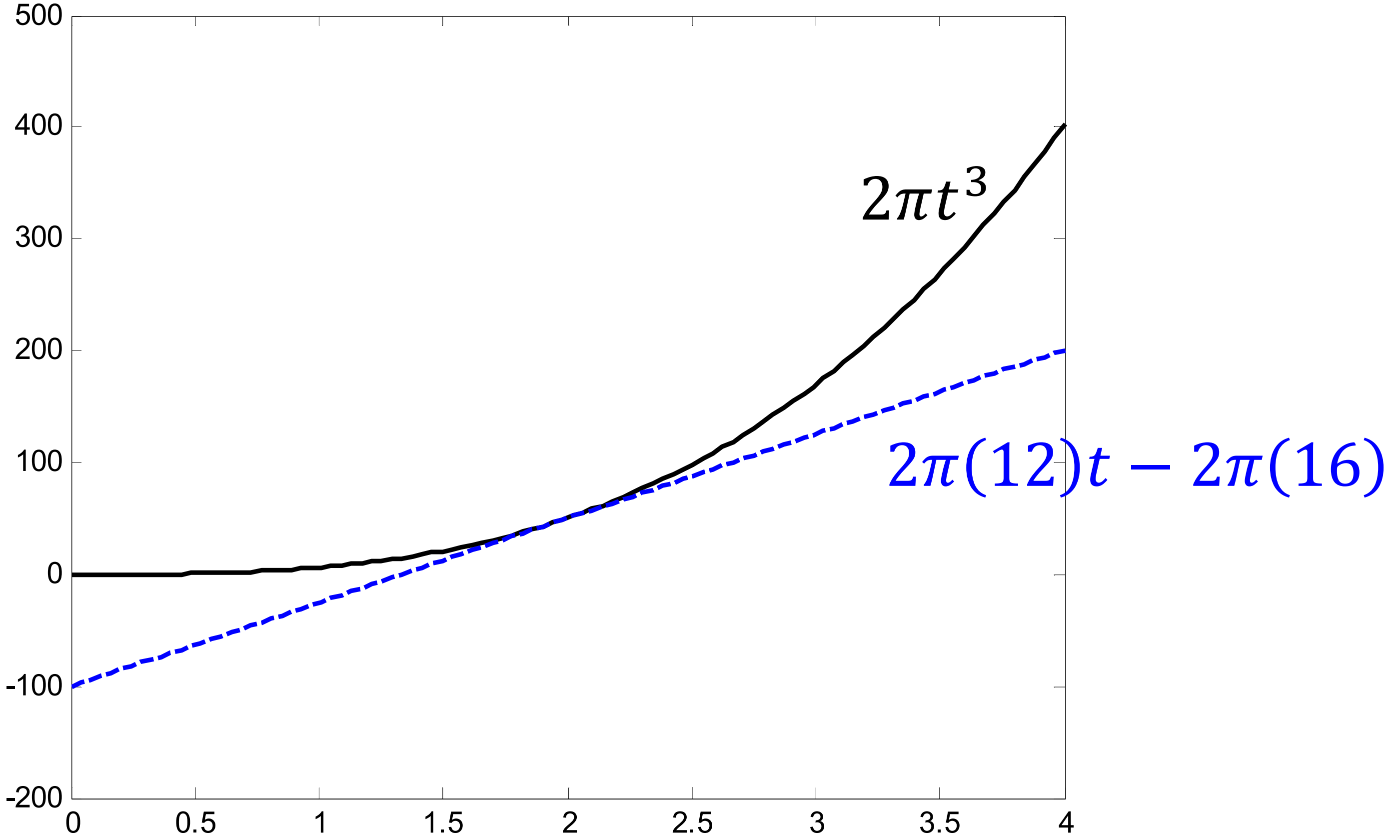
$$\cos\left(\phi(t)\right) \approx \cos\left(\phi'(t_0)t + \phi(t_0) - t_0\phi'(t_0)\right)$$

  - Now, we can directly compare the terms with $\cos(2\pi \boldsymbol{f_0} t + \phi)$.

First-order (straight-line) approximation/linearization

- For example, for t near $t = 2$,

$$2\pi t^3 \approx 2\pi\left(3t^2\right)\Big|_{t=2}(t-2) + 2\pi t^3\Big|_{t=2} = 2\pi(12)t - 2\pi(16)$$



First-order (straight-line) approximation/linearization

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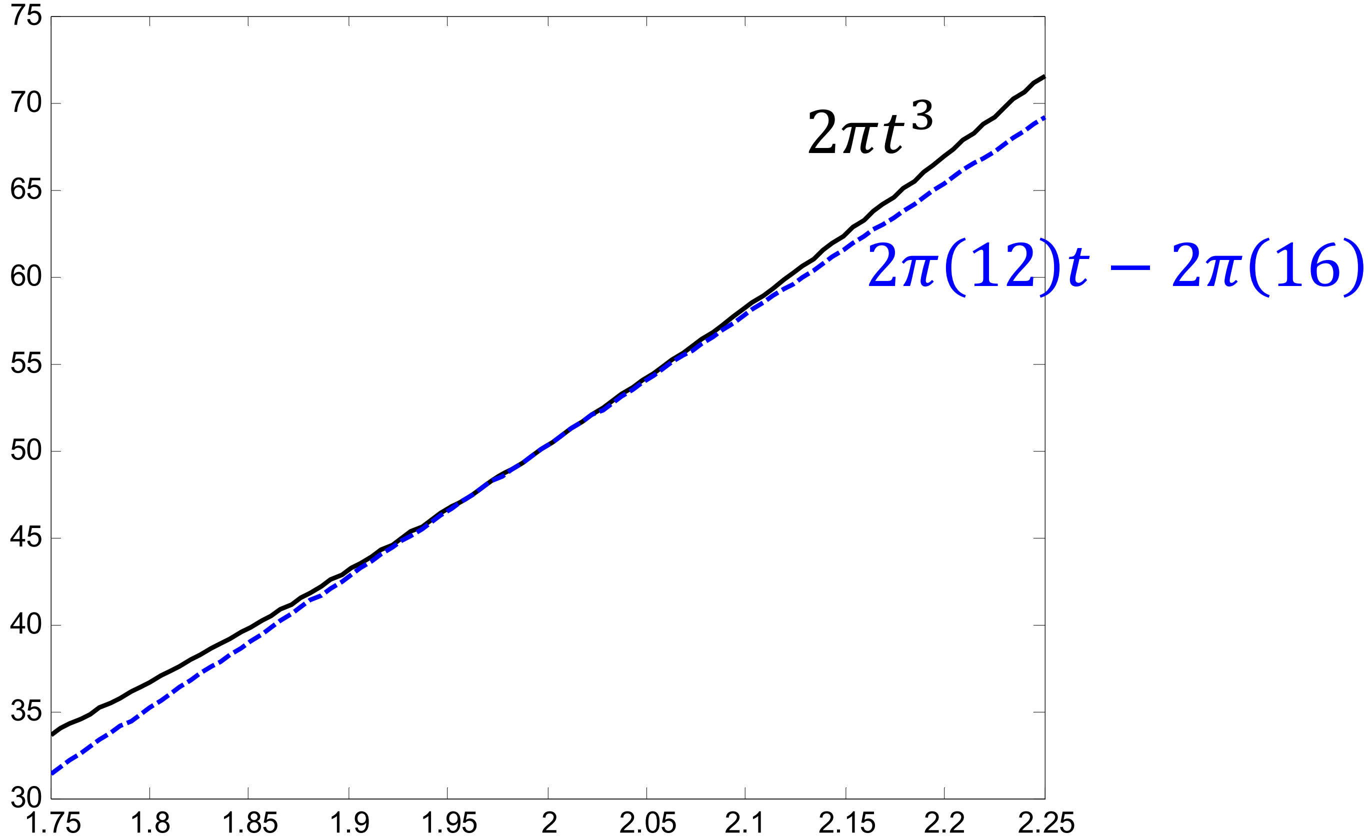

First-order (straight-line) approximation/linearization

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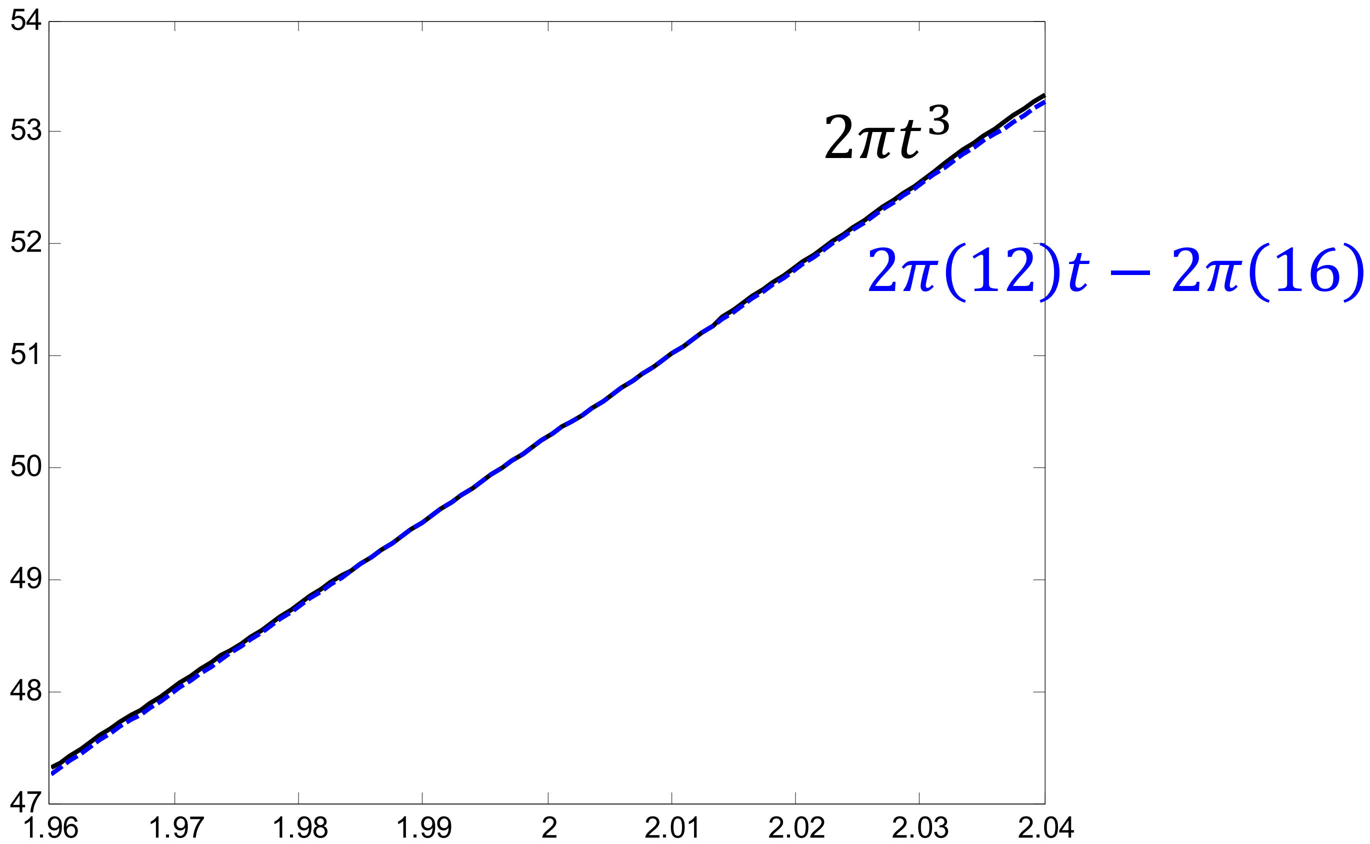


Same idea

- Suppose we want to find $\sqrt{15.9}$.
- Let $g(x) = \sqrt{x}$.
  - Note that $\frac{d}{dx} g(x) = \frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$.
- Approximation: $g(x) \approx g'(x_0)(x - x_0) + g(x_0)$
- 15.9 is near 16.
- $\sqrt{15.9} = g(15.9)$
  - $\approx g'(16)(15.9 - 16) + g(16)$
  - $= \frac{1}{2\sqrt{16}}(-0.1) + \sqrt{16} = -\frac{0.1}{8} + 4 = 3.9875$
- MATLAB:

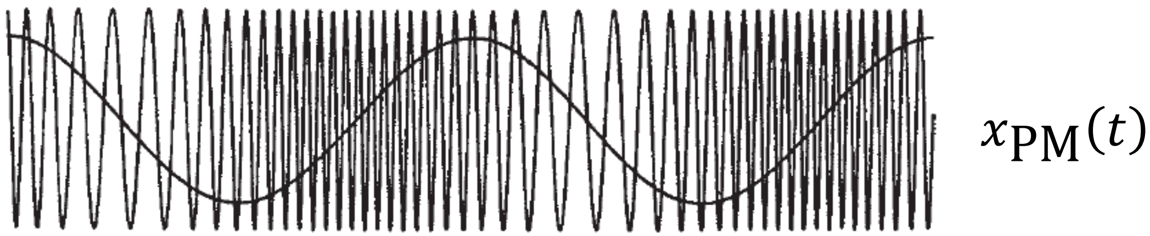
```
>> sqrt(15.9)
ans =
   3.987480407475377
```

Phase Modulation

Figure 24

(a)

When $m(t)$ and hence the phase of $x_{\mathrm{PM}}(t)$ change **continuously**, it is difficult to see the connection with the actual plot of $x_{\mathrm{PM}}(t)$.

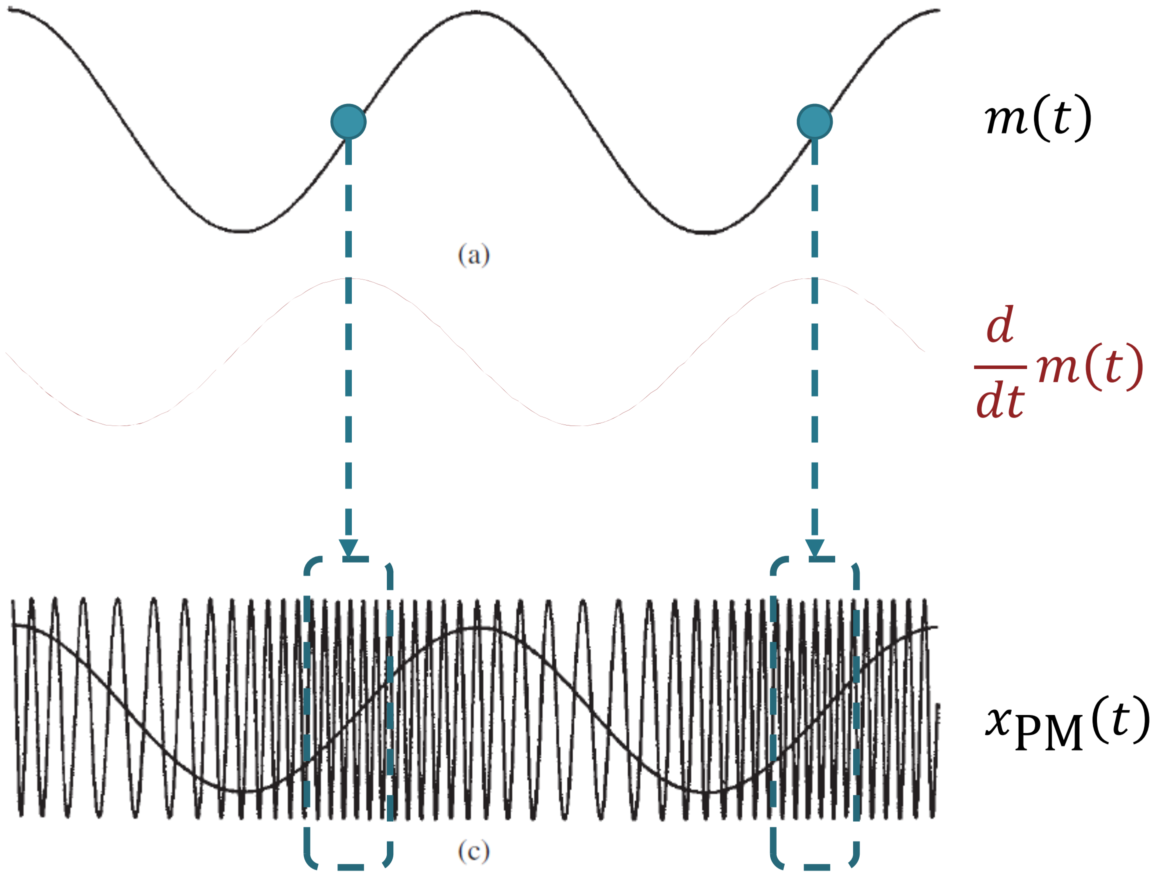


(c)

New Fact: In $x_{\mathrm{PM}}(t)$, the **instantaneous frequency** varies in proportion with the **slope** of $m(t)$.

Phase Modulation

Figure 24

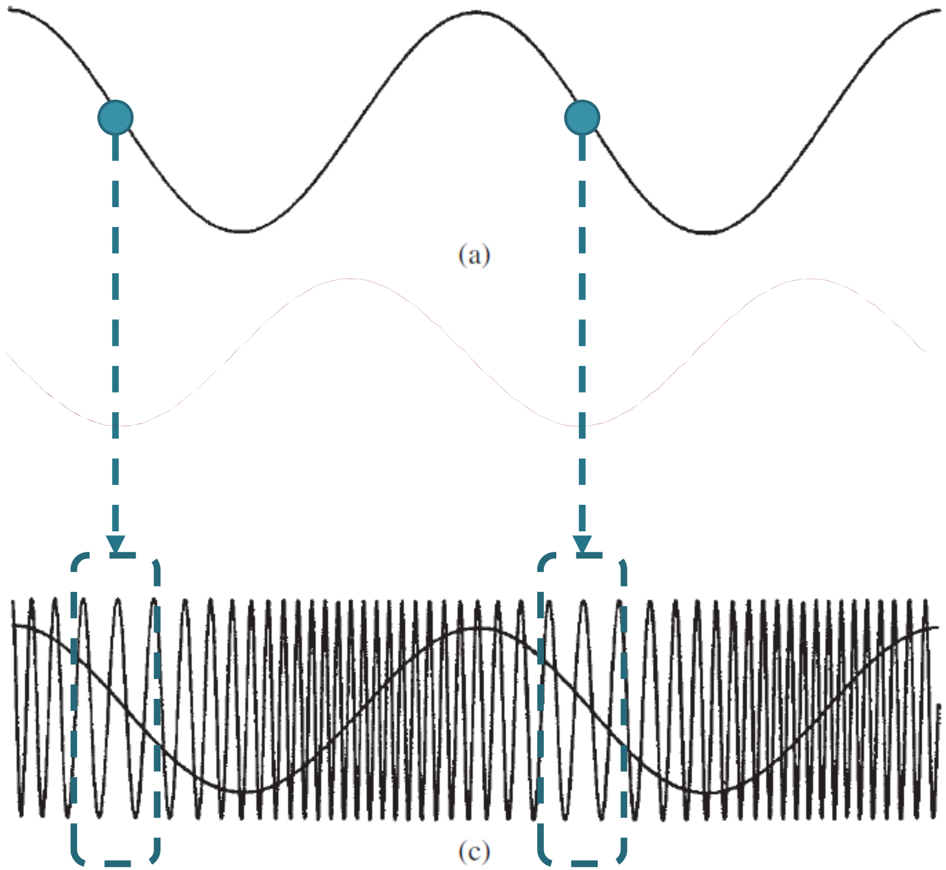


The time at which the **slope** of $\boldsymbol{m(t)}$ is at its **maximum** value corresponds to the time at which $\boldsymbol{x_{PM}(t)}$ has **maximum frequency**.

New Fact: In $x_{PM}(t)$, the **instantaneous frequency** varies in proportion with the **slope** of $m(t)$.

Phase Modulation

Figure 24



$m(t)$

$\frac{d}{dt}m(t)$

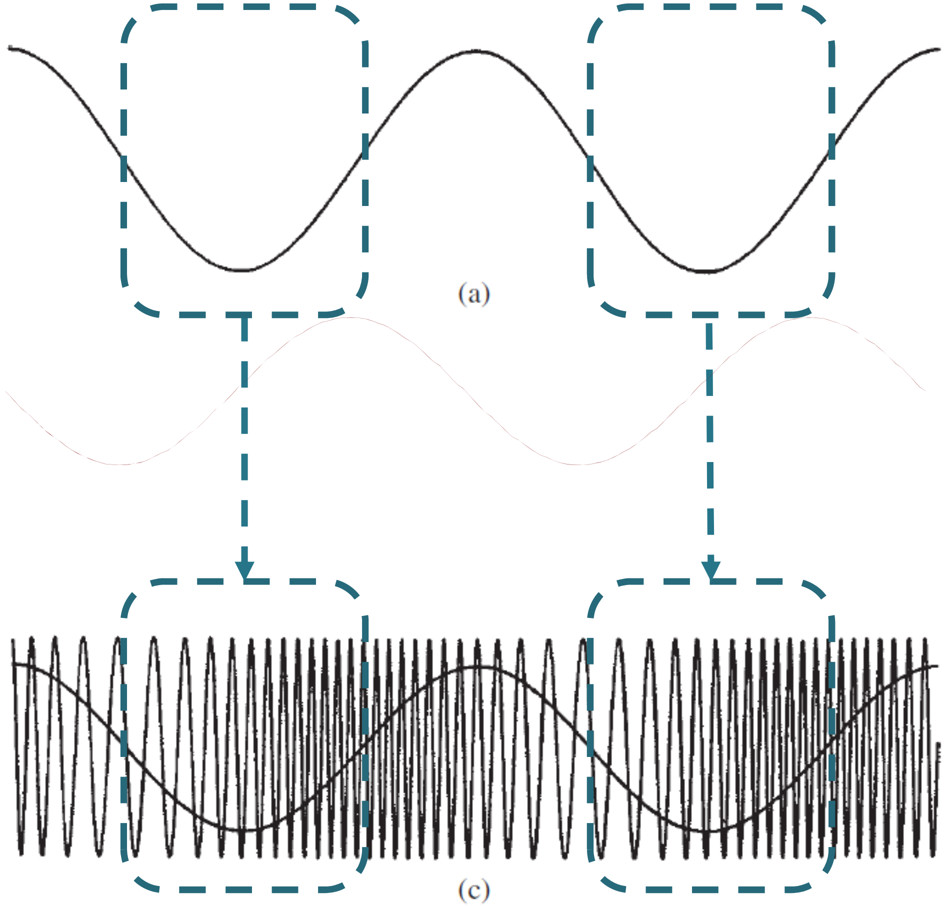
$x_{\text{PM}}(t)$

The time at which the **slope** of $\boldsymbol{m(t)}$ is at its **minimum** value corresponds to the time at which $\boldsymbol{x_{\text{PM}}(t)}$ has **minimum frequency**.

New Fact: In $x_{\text{PM}}(t)$, the **instantaneous frequency** varies in proportion with the **slope** of $m(t)$.

Phase Modulation

Figure 24



$m(t)$

$\frac{d}{dt} m(t)$

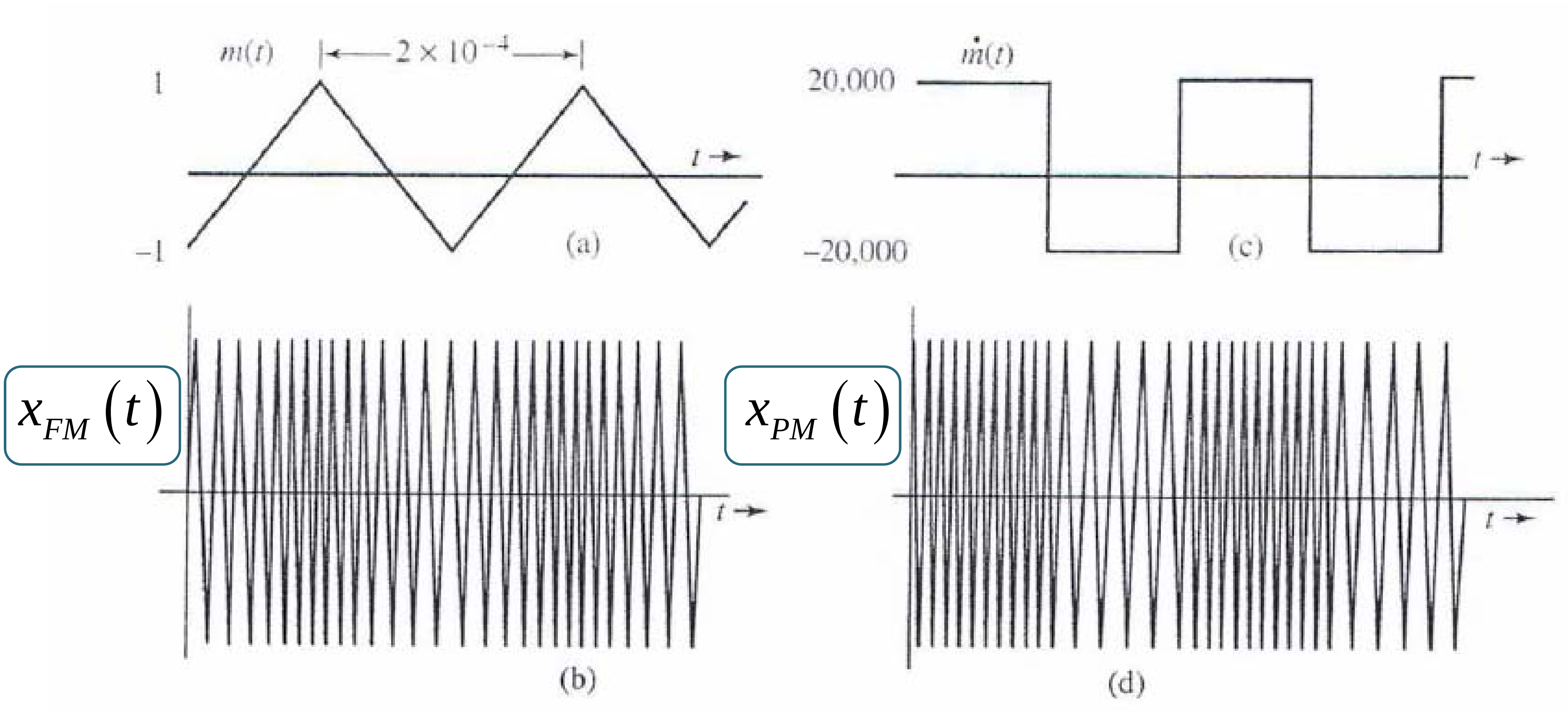
$x_{\text{PM}}(t)$

The time interval during which the **slope** of $\boldsymbol{m(t)}$ is **increasing** corresponds to the time interval during which $\boldsymbol{x_{\text{PM}}(t)}$ has **increasing frequency**.

New Fact: In $x_{\text{PM}}(t)$, the **instantaneous frequency** varies in proportion with the **slope** of $m(t)$.

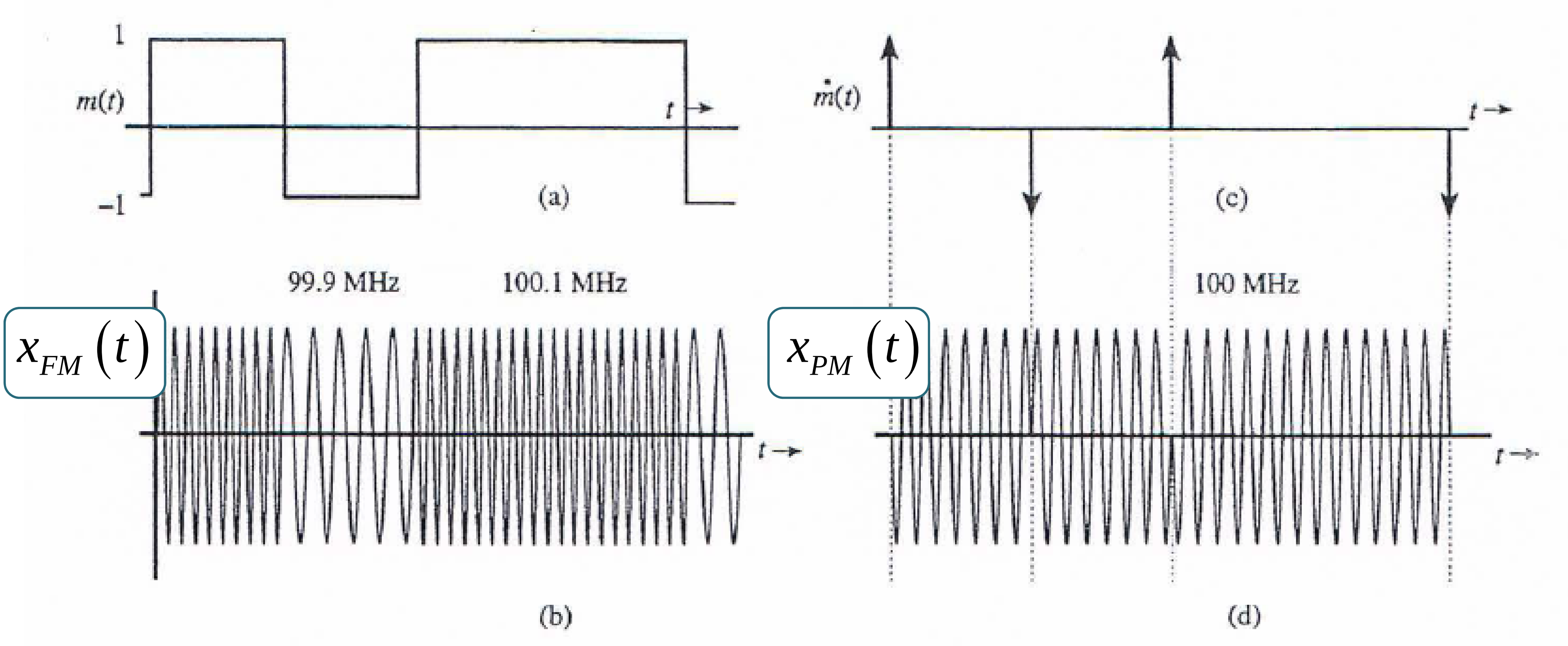
FM vs. PM

Figure 28

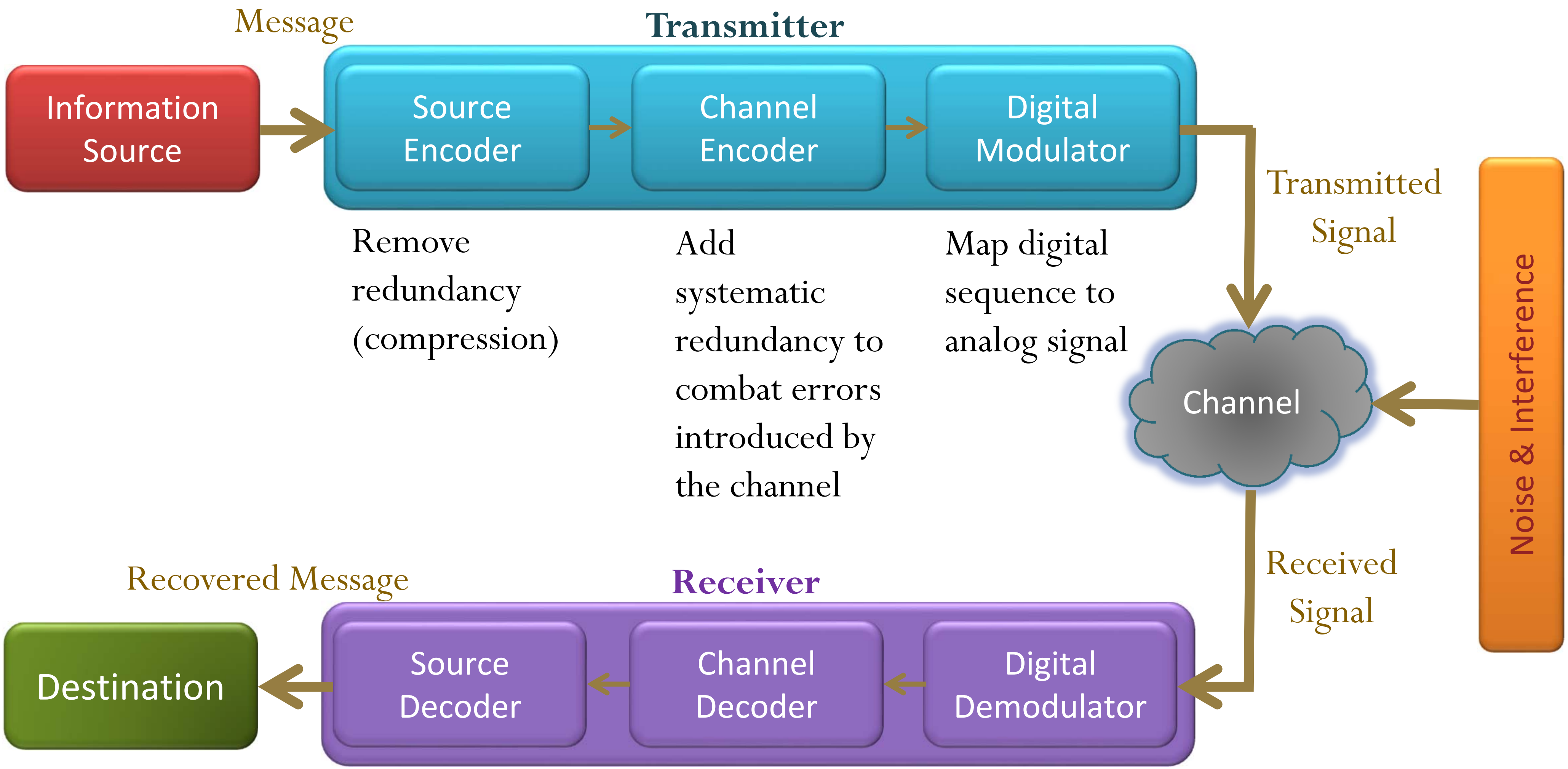


Remark: To see $x_{PM}(t)$ of time varying m(t), it is usually easier to look at the instantaneous freq. via the derivative first.

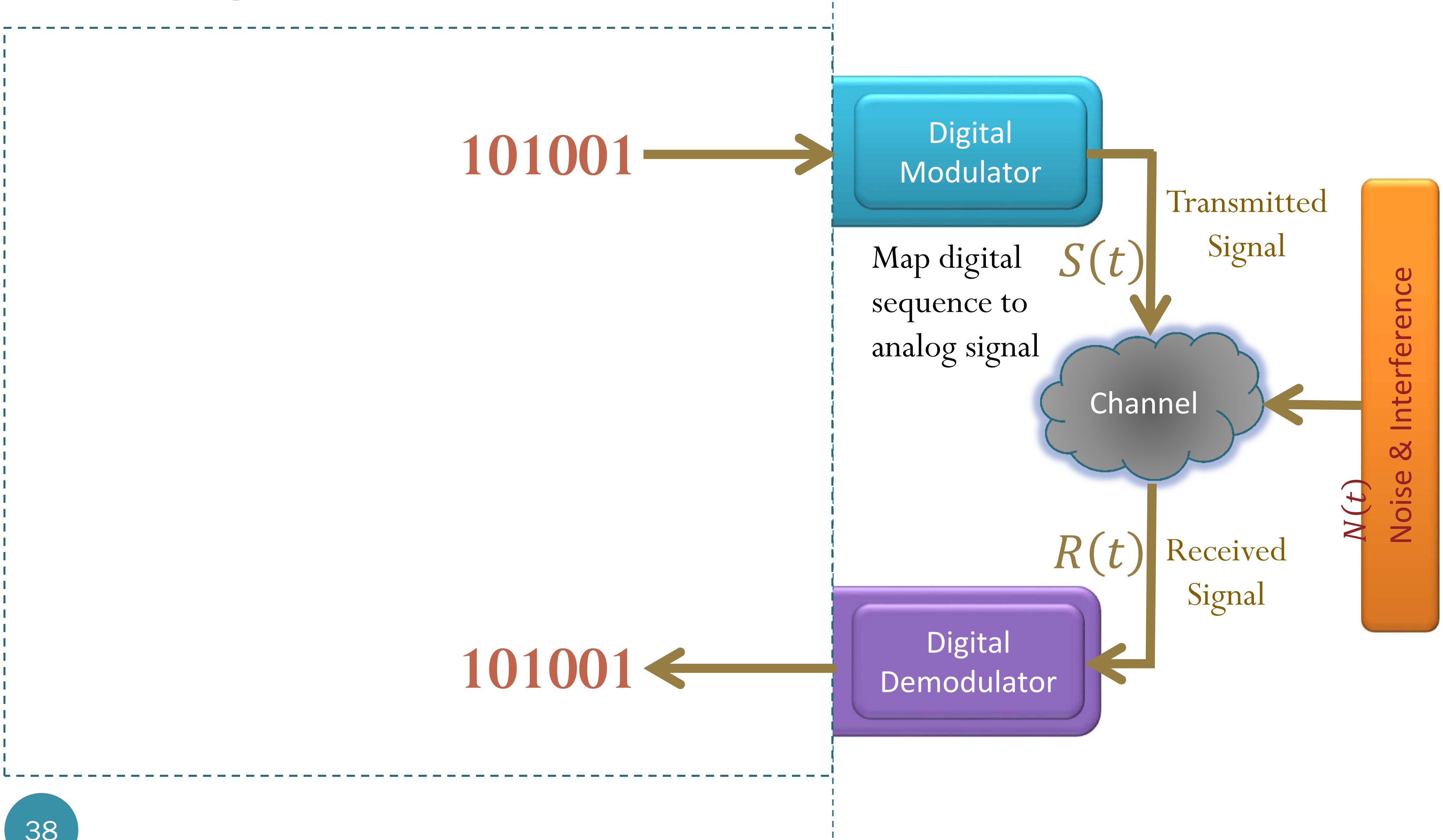
FM vs. PM



Elements of digital commu. sys.



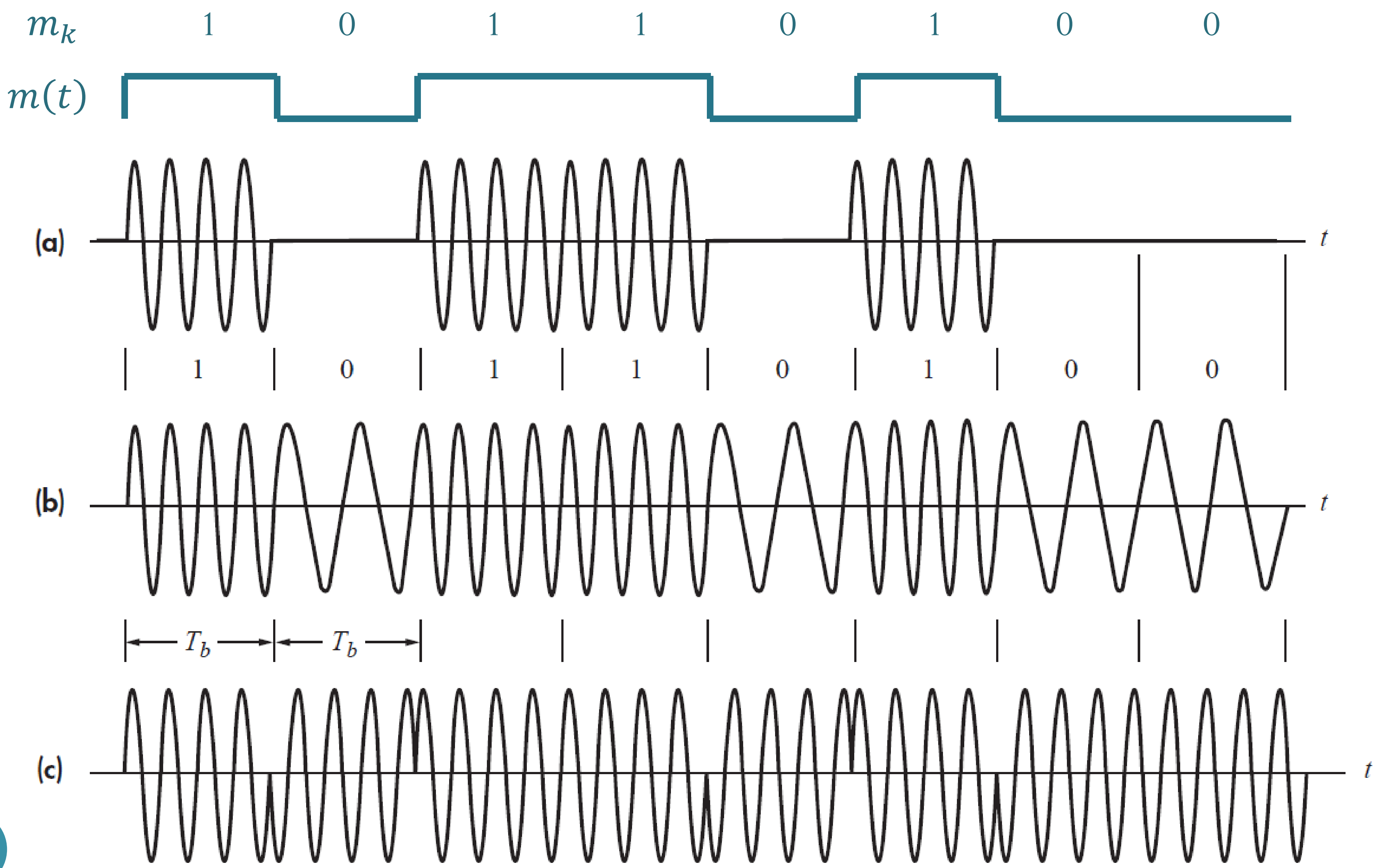
Digital Modulation/Demodulation



Digital Version of

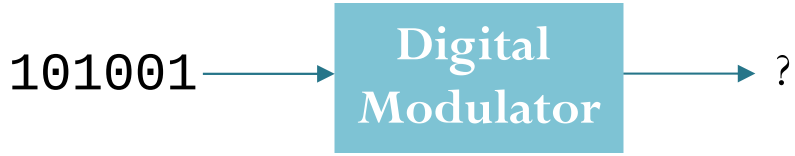
- Use digital signal to modulate the amplitude, frequency, or phase of a sinusoidal carrier wave.
  - Think of $m(t)$ as a train of scaled (rectangular) pulses.
  - The modulated parameter will be switched or keyed from one discrete value to another.
- Three basic forms:
  - amplitude-shift keying (ASK)
  - frequency-shift keying (FSK)
  - phase-shift keying (PSK)

Binary ASK, FSK, and PSK

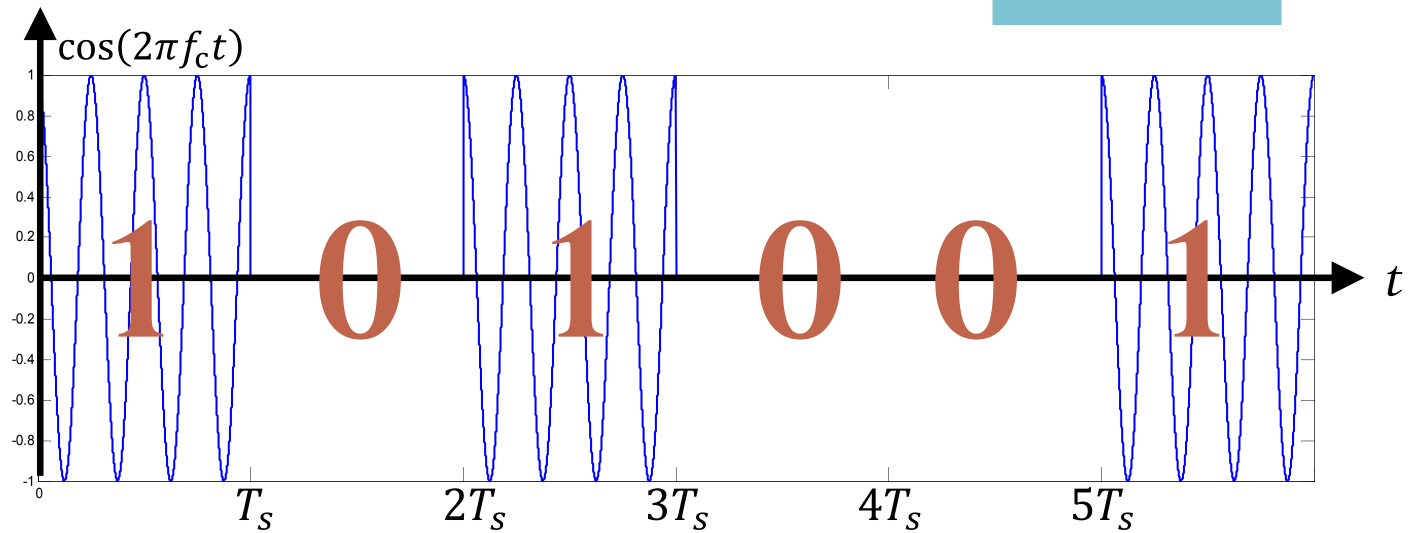


[Carlson & Crilly, 2009, Fig 14.1-1 p 649]

Simple ASK: ON-OFF Keying (OOK)



$f_c = 4$ Hz
Bit rate = 1 bps

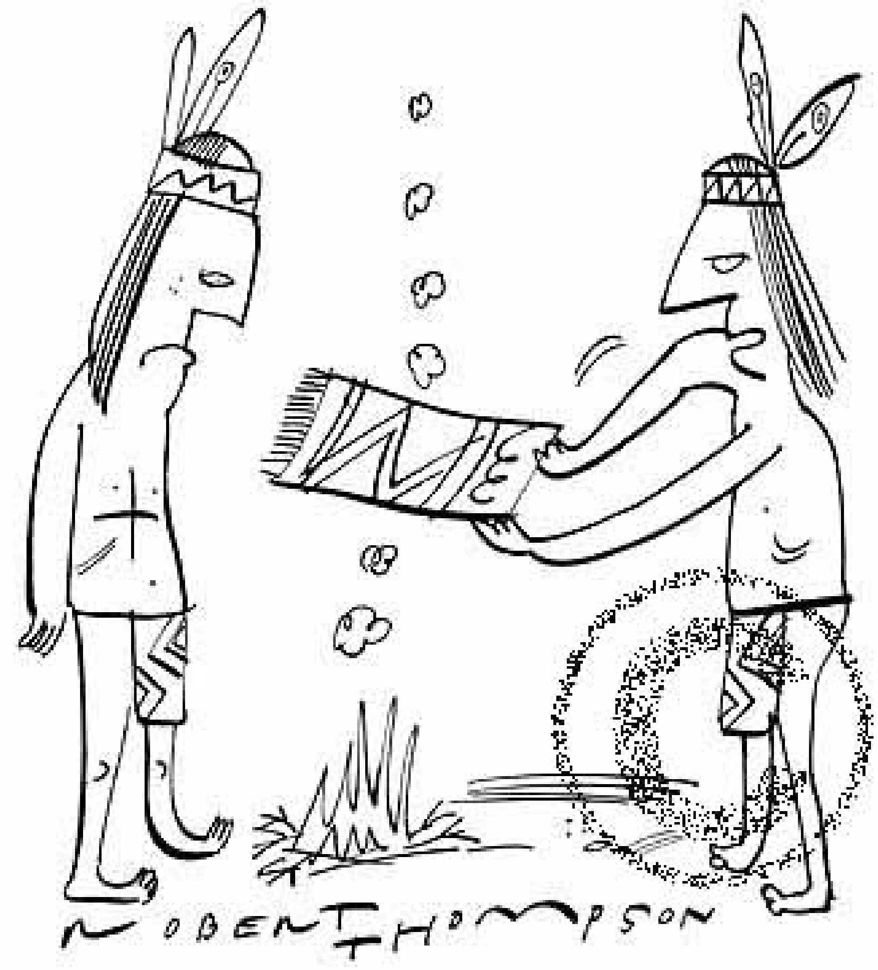


t

[ASK_playTones_Demo.m]

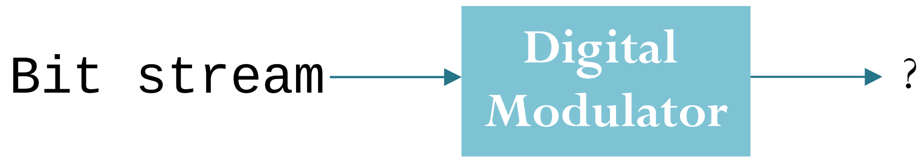
Simple “ASK”: “ON-OFF Keying”

Smoke signal

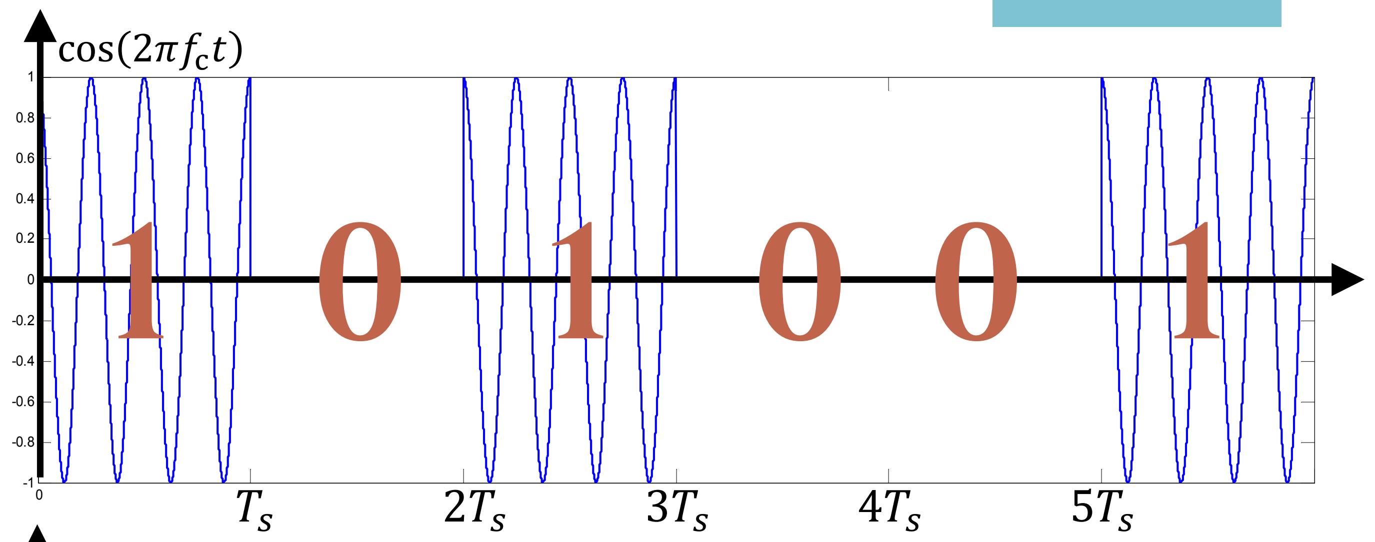


"It's no use the signal's too weak."

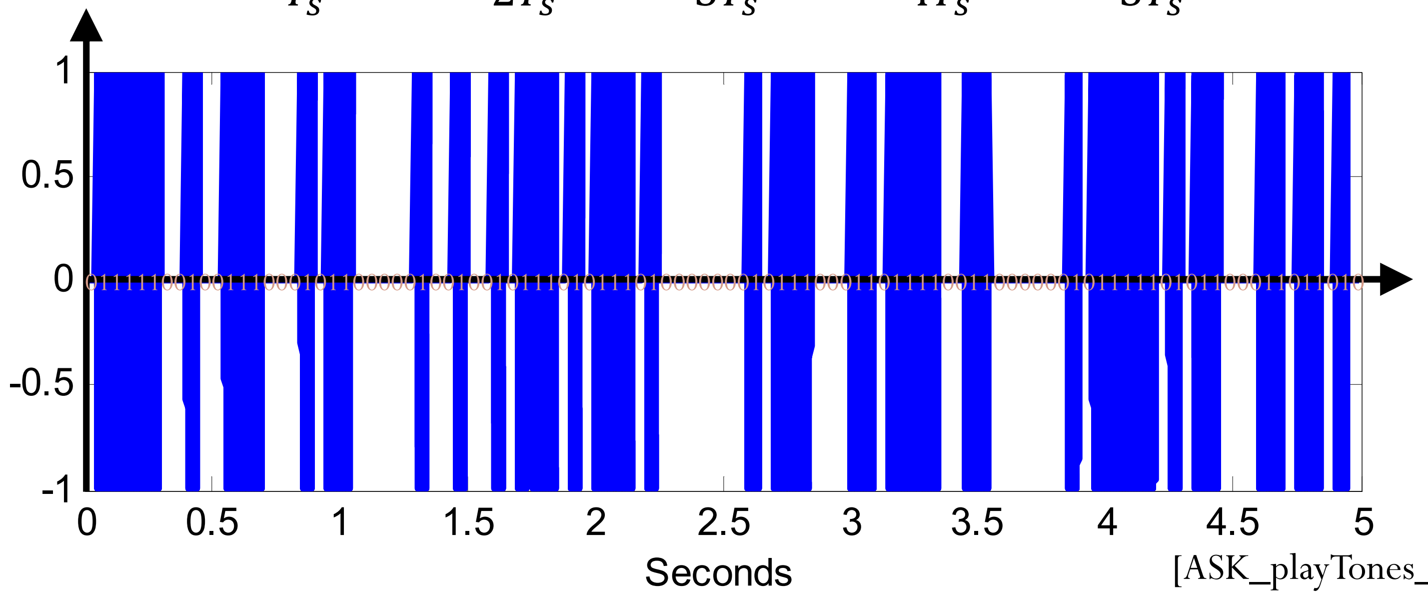
Simple ASK: ON-OFF Keying (OOK)



$f_c = 4$ Hz
Bit rate = 1 bps

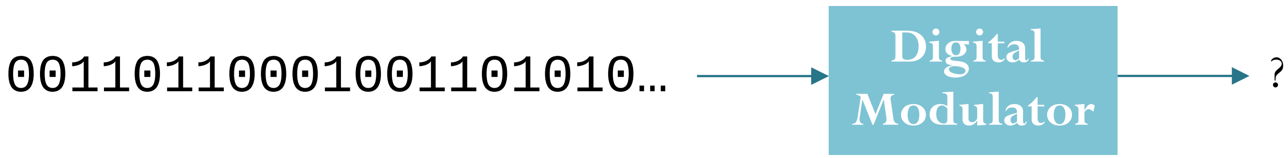


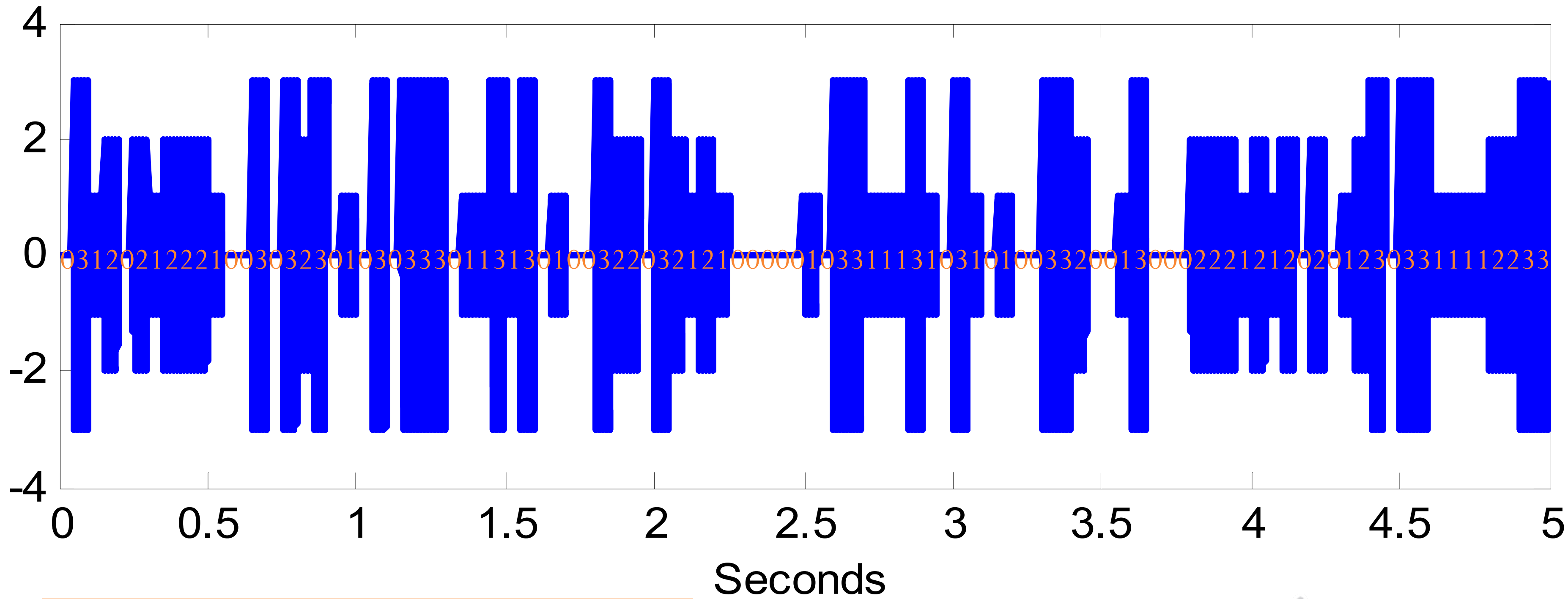
$f_c = 100$ Hz
Bit rate = 20 bps



[ASK_playTones_Demo.m]

ASK: Higher Order Modulation



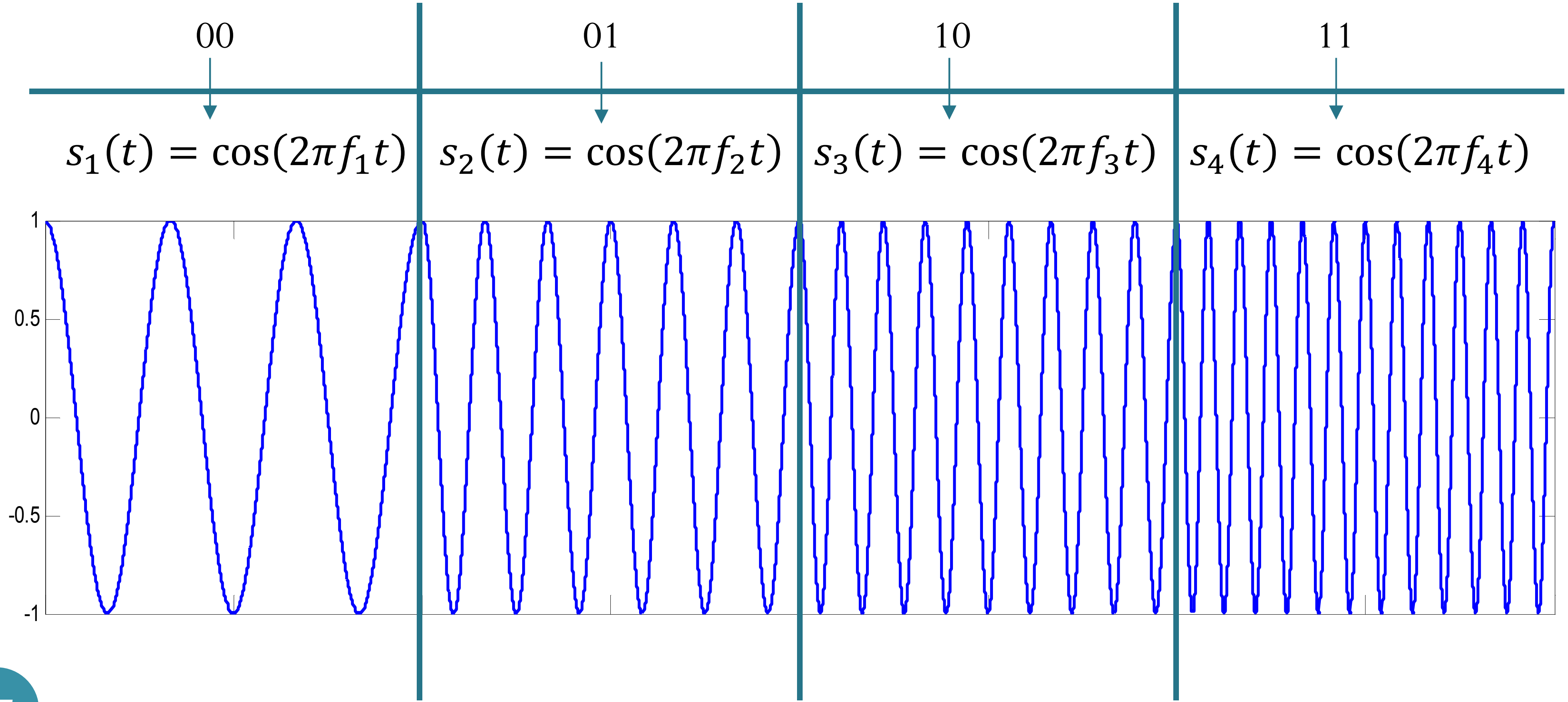


$f_c = 100$ Hz
Symbol rate = 20 symbols per second
Bit rate = 40 bps

FSK

$M = 4$

$f_c \in \{ f_1, f_2, f_3, f_4 \} = \{3, 6, 9, 12\}$ [Hz]



[FSK_playTones_Demo.m]

FSK

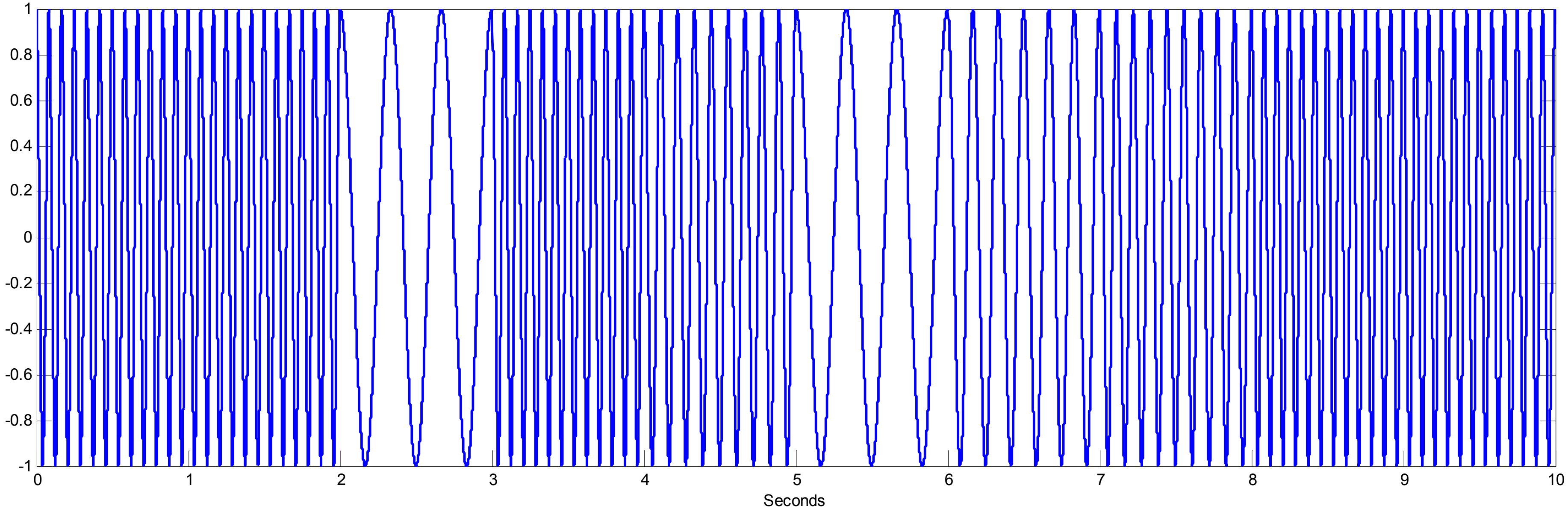
11110011100001101111 → **Digital Modulator** → ?

M = 4 $f_c \in \{f_1, f_2, f_3, f_4\} = \{3, 6, 9, 12\}$ [Hz]

[11 11 00 11 10 00 01 10 11 11]

↓

$f_c =$ [12 12 3 12 9 3 6 9 12 12] Hz



[FSK_playTones_Demo.m]

FSK

11110011100001101111 → **Digital Modulator** → ?

M = 4

$f_c \in \{f_1, f_2, f_3, f_4\} = \{100, 200, 300, 400\}$ [Hz]

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[11   11   00   11   10   00   01   10   11   11]
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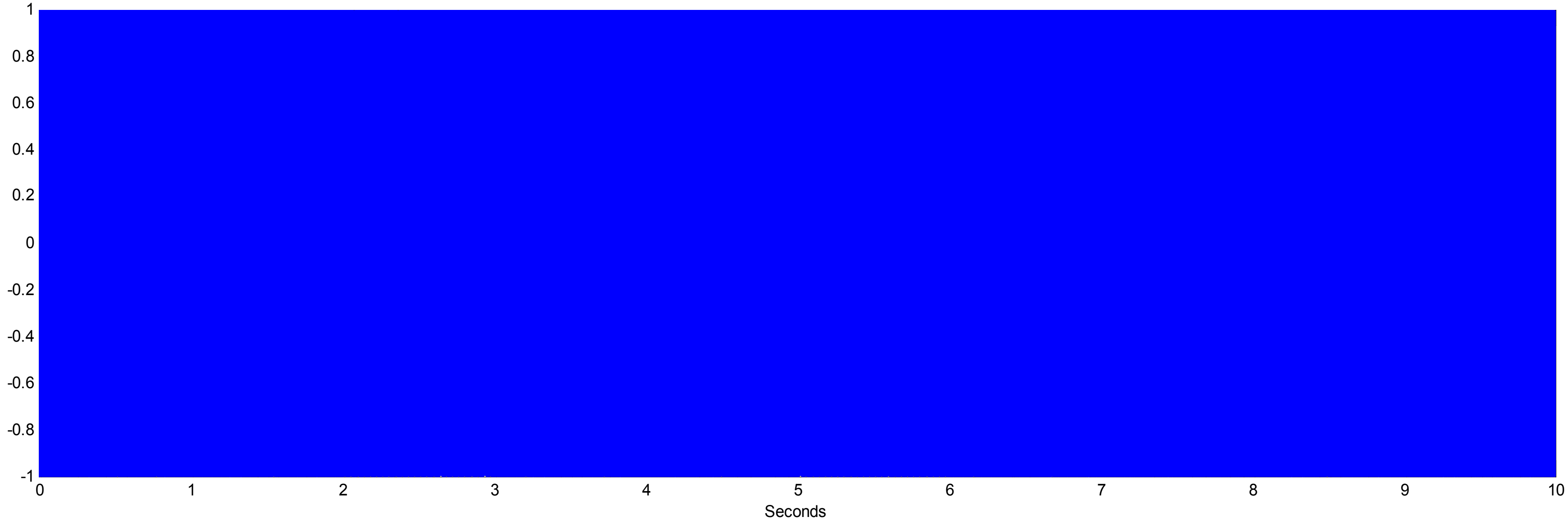
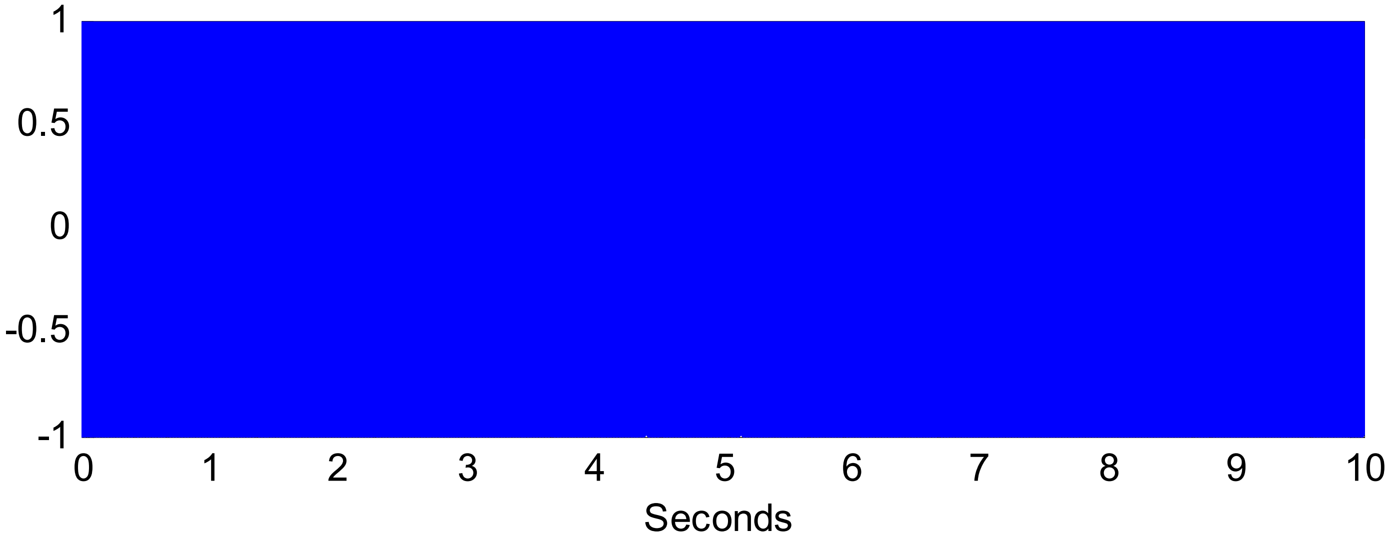
$f_c =$ [400 400 100 400 300 100 200 300 400 400] Hz

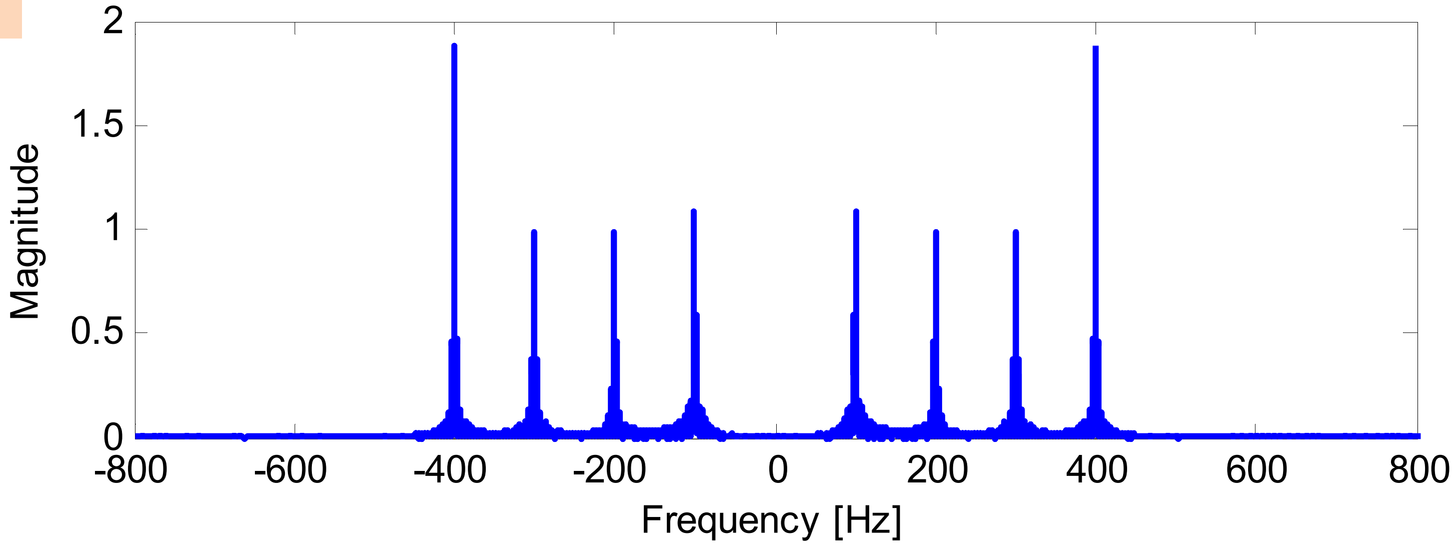


[FSK_playTones_Demo.m]

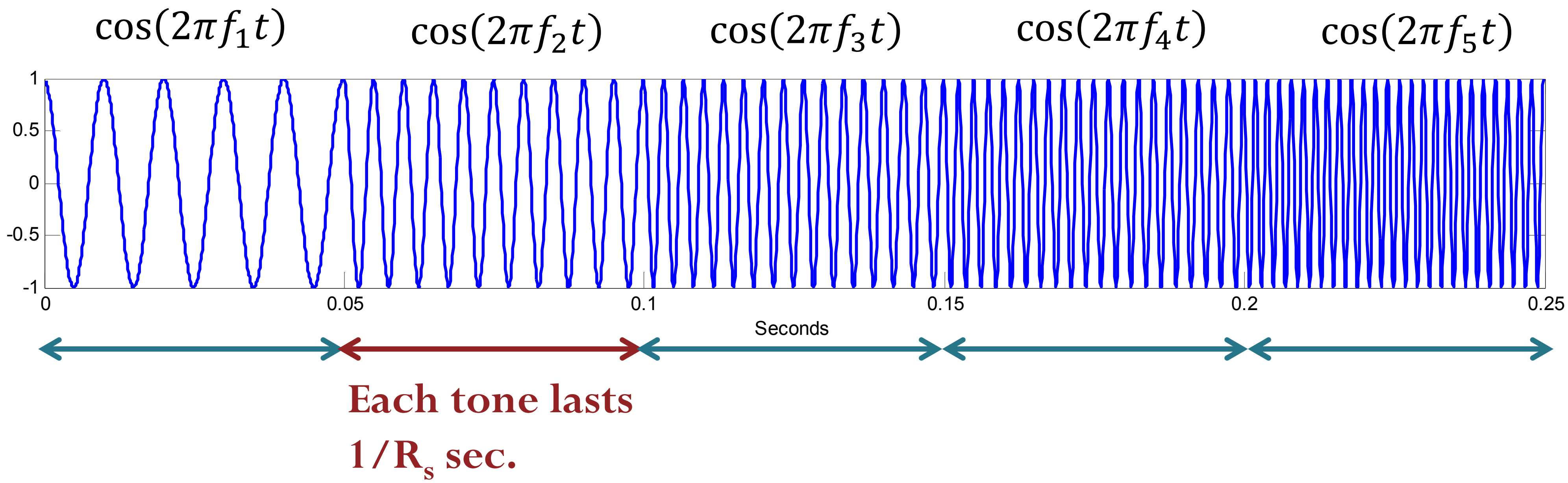
FSK



$R_s = 5$
Bit rate = 10 bps



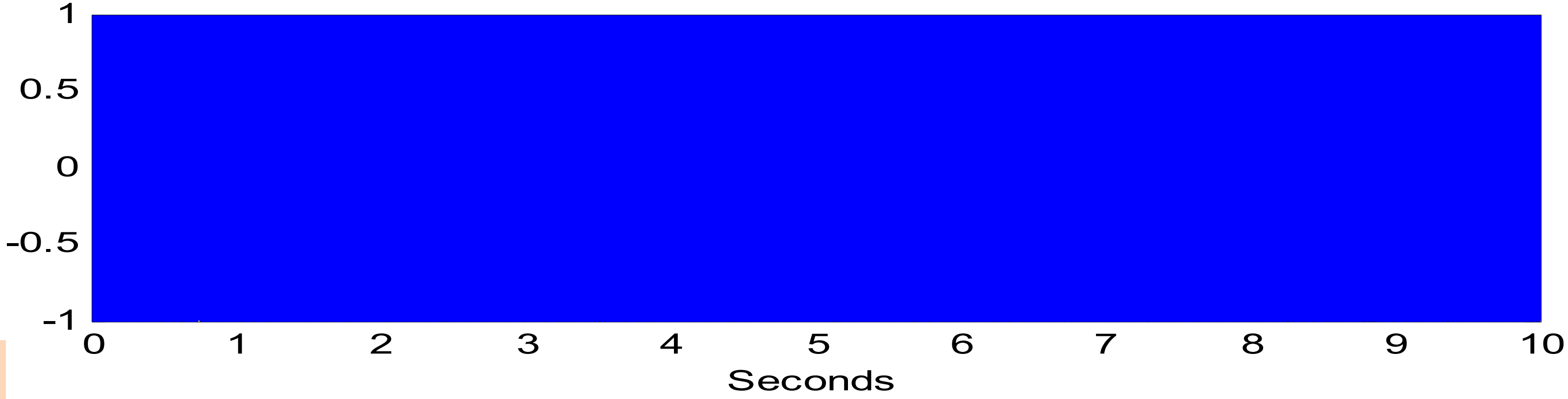
Five Frequencies



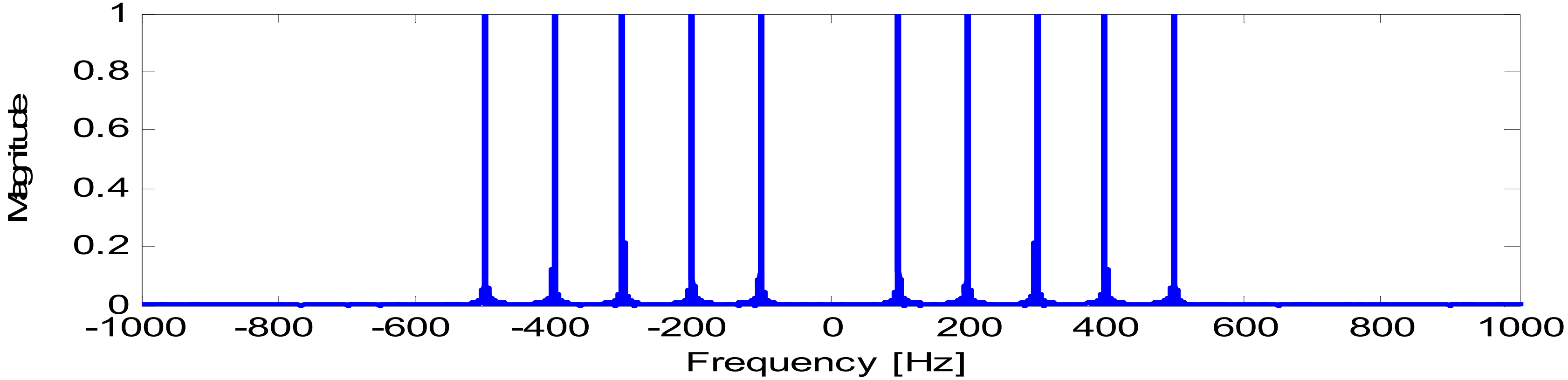
Rate = R_s frequency-changes per second

Spectrum of Five Frequencies (1/5)

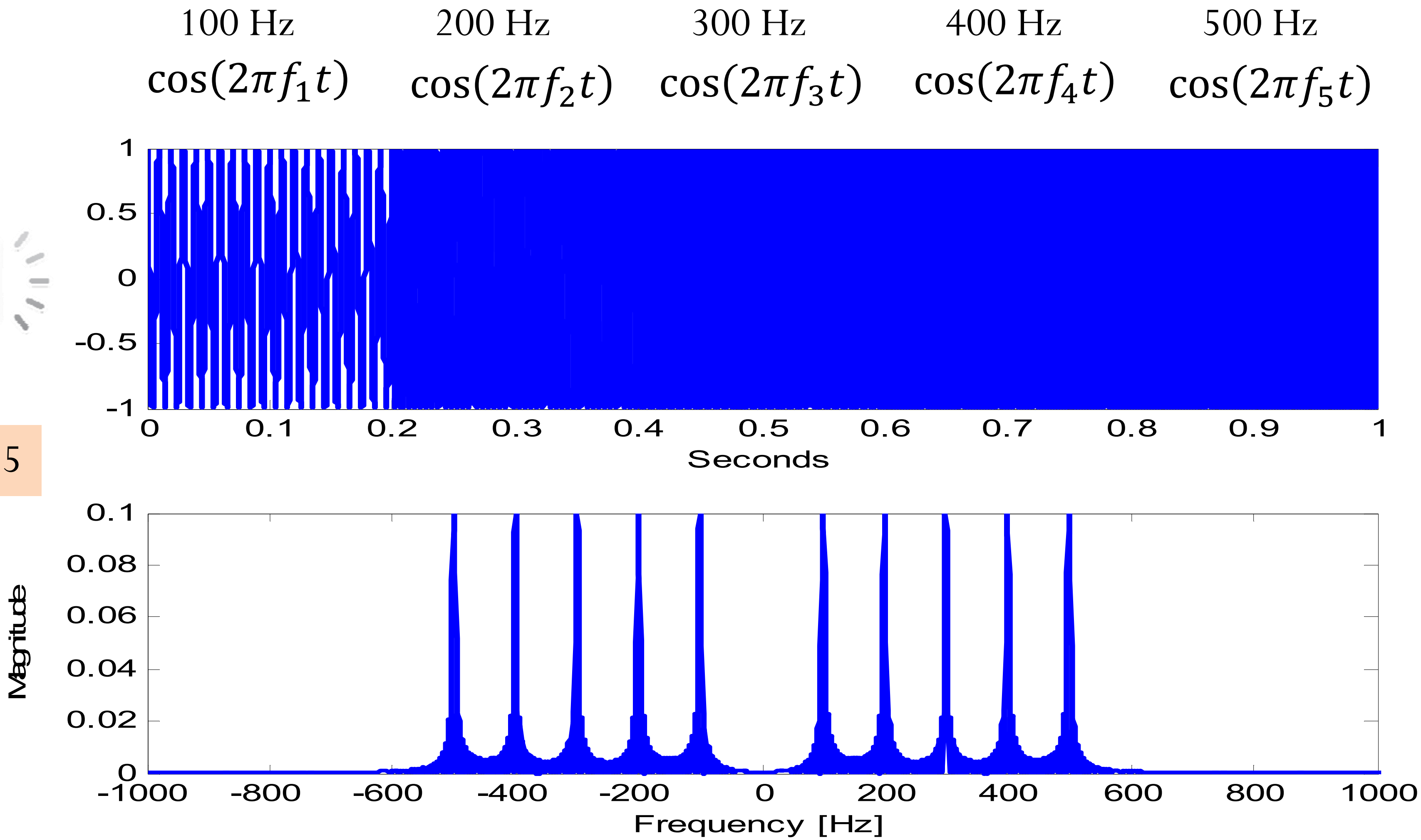
100 Hz	200 Hz	300 Hz	400 Hz	500 Hz
$\cos(2\pi f_1 t)$	$\cos(2\pi f_2 t)$	$\cos(2\pi f_3 t)$	$\cos(2\pi f_4 t)$	$\cos(2\pi f_5 t)$



$R_s = 0.5$



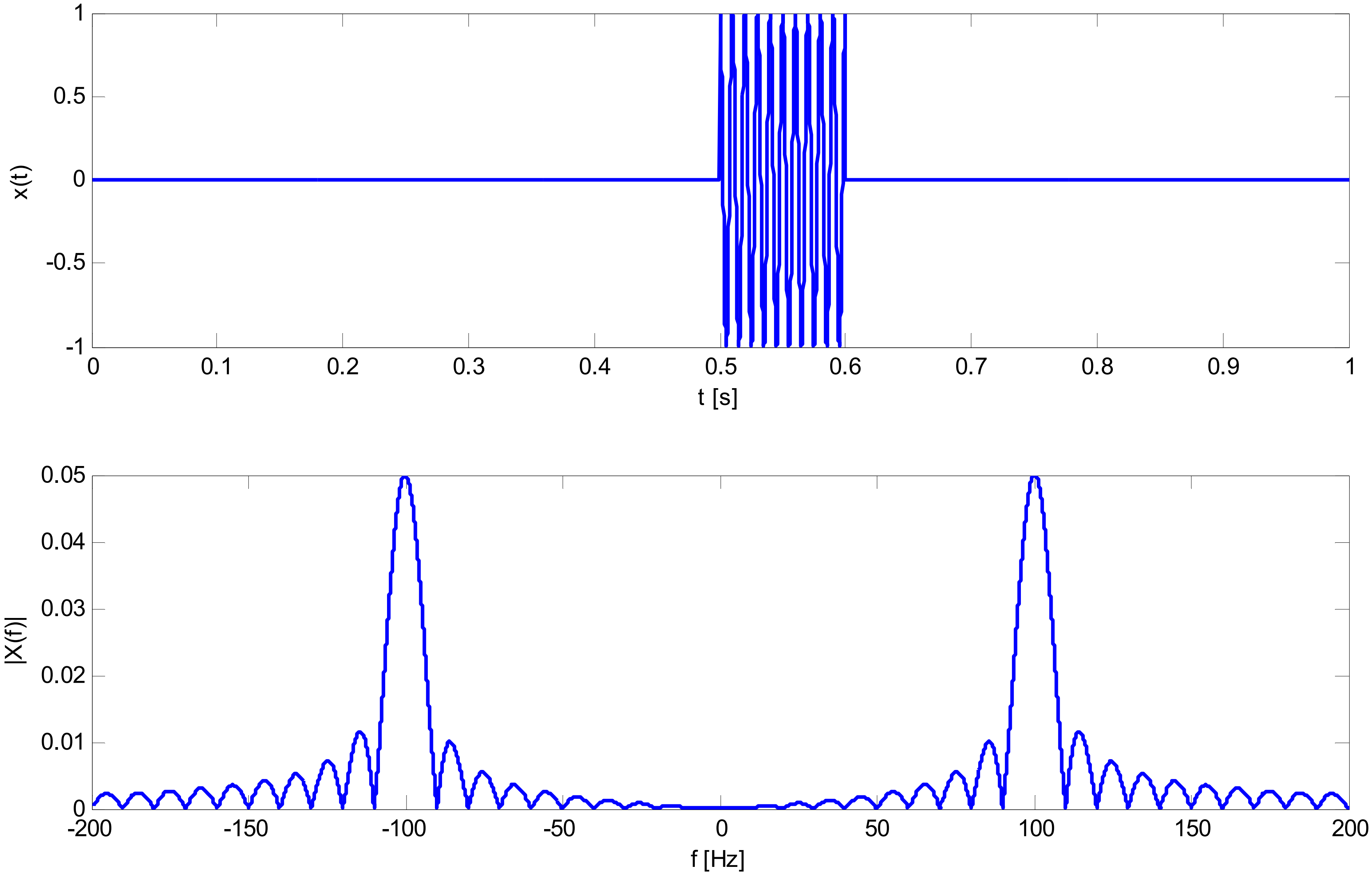
Spectrum of Five Frequencies (2/5)



$R_s = 5$

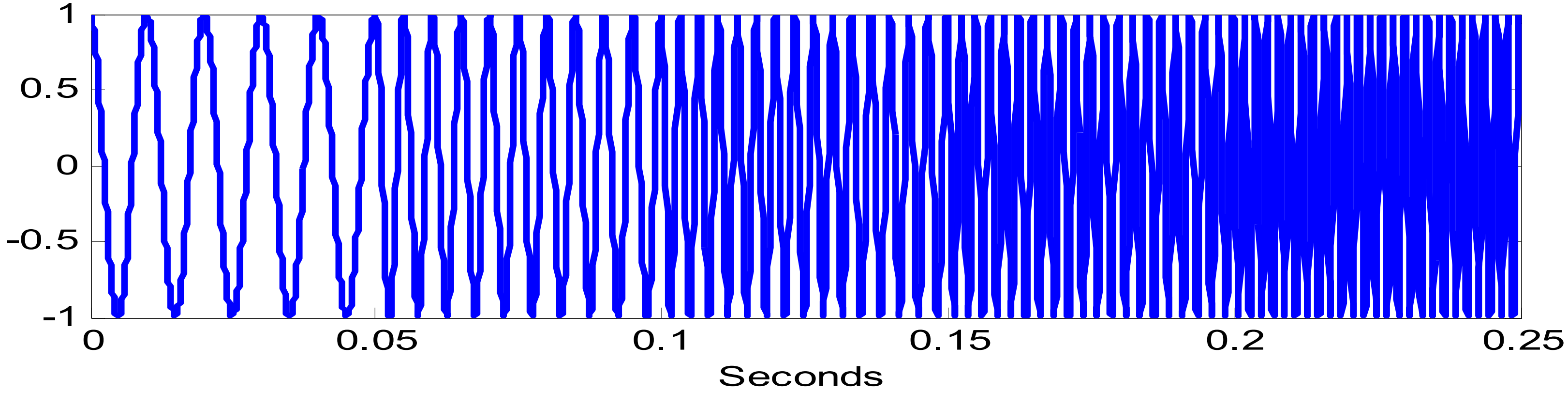
Cosine Pulse

$$x(t) = \begin{cases} \cos(2\pi(100)t), & 0.5 \le t \le 0.6, \\ 0, & \text{otherwise.} \end{cases}$$

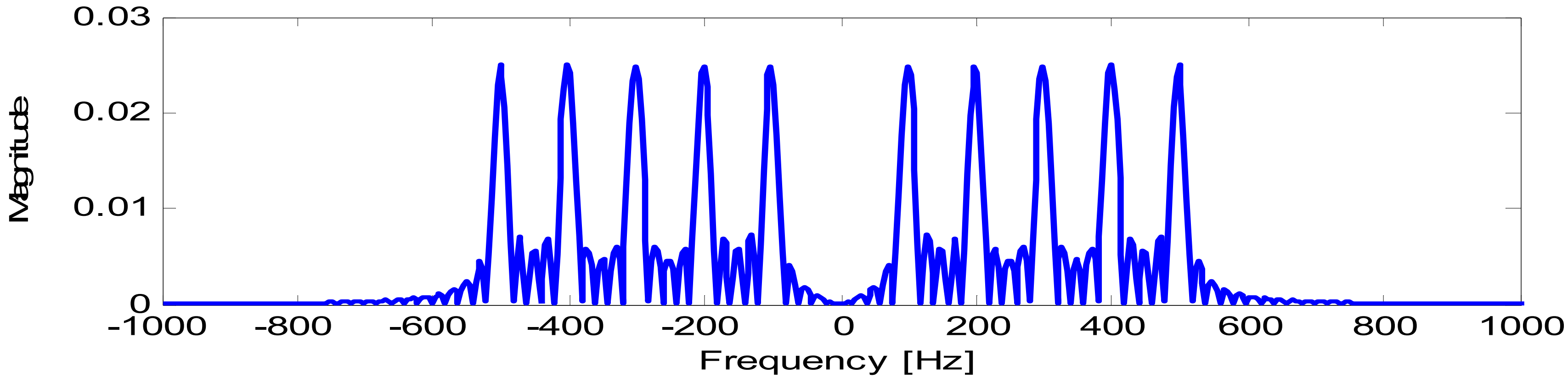


Spectrum of Five Frequencies (3/5)

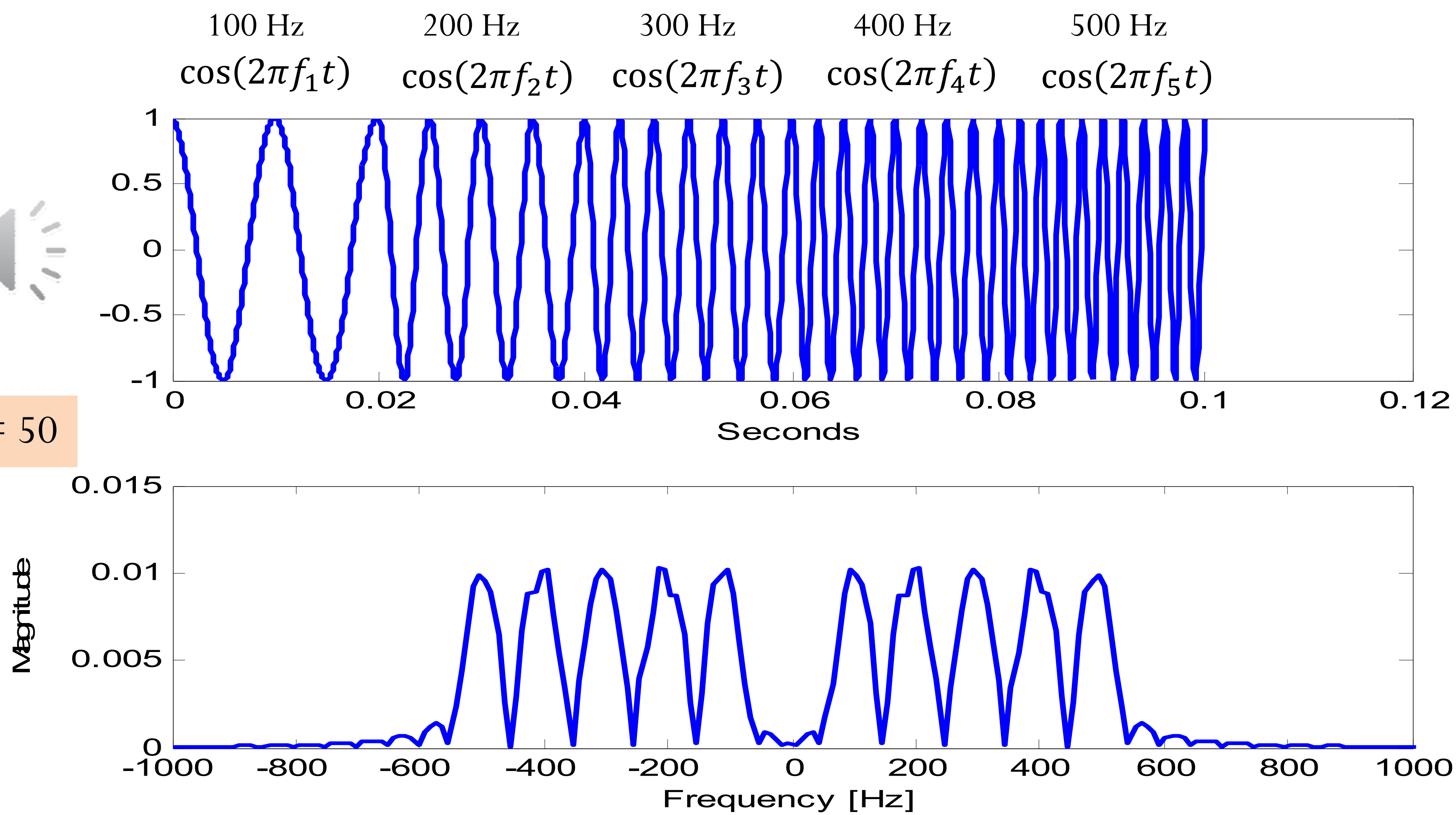
100 Hz	200 Hz	300 Hz	400 Hz	500 Hz
$\cos(2\pi f_1 t)$	$\cos(2\pi f_2 t)$	$\cos(2\pi f_3 t)$	$\cos(2\pi f_4 t)$	$\cos(2\pi f_5 t)$



$R_s = 20$



Spectrum of Five Frequencies (4/5)



$R_s = 50$

Spectrum of Five Frequencies (5/5)

100 Hz	200 Hz	300 Hz	400 Hz	500 Hz
$\cos(2\pi f_1 t)$	$\cos(2\pi f_2 t)$	$\cos(2\pi f_3 t)$	$\cos(2\pi f_4 t)$	$\cos(2\pi f_5 t)$

$R_s = 0.5$